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# MAXIMUM STRESSES

IN

# FRAMED BRIDGES.

BY

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# PREFACE.

THIS treatise is clearly a proper supplement to the ordinary works on strains in trusses. Every case of examination into causes of failure of broken structures furnishes substantial evidence that such a treatise is an important addition to the literature heretofore published.

The thoroughly practical character of all of Prof. Cain's literary works, and the flattering reception of his previous work (*Voussoir Arches*) by working engineers, have induced the publisher to reprint these recent contributions to the Magazine in the Science Series.

(ED. OF MAGAZINE).

# MAXIMUM STRESSES
IN
# FRAMED BRIDGES.

THE writer has made the endeavor in the following pages to investigate, for the live loads assumed, the maximum stresses that can ever occur in the chords, as well as in the web members of a bridge; also the most economical height of trusses. In so doing he has necessarily gone over some old ground; in the briefest manner, however, consistent with logical development; and has compared, approximately, some leading American types of bridges as regards weight. The unit strains were determined by the modification of Launhardt's formula, proposed by the writer in the November, 1877, number of this Magazine, which, it was thought, was peculiarly adapted

to the comparison of trusses, besides illustrating the "new method" of designating "Structures of Iron and Steel," which may possess interest at this time.

1. A Framed Bridge is generally composed of two or more *trusses* or *frames*, as A*d*B, Fig. 1; which lie in vertical

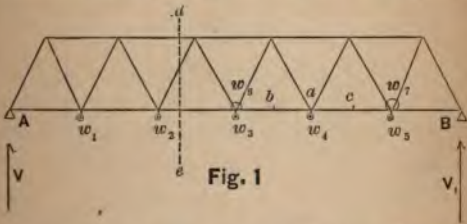


Fig. 1

planes, and are connected together by *bracing*, including the *floor beams*, on which longitudinal *stringers* rest, which support the cross ties and rails of a railroad bridge or the flooring of a highway bridge.

The upper and lower horizontal members of a truss are called respectively, the upper and lower *chords*, the bracing between them the *web*. The members of *the web that* act always as ties are called

*main ties;* those acting always as struts, *main braces* or *posts;* and those members that act alternately as ties and struts are called *counters*—a term likewise applied to pieces that are not strained appreciably by the dead load or any uniform load on the structure, but are strained when the live load is distributed in a certain manner.

2. As the roadway is supported by the top or lower chord, the bridge is called a *deck* bridge or a *through* bridge. The intersection as *a*, Fig. 1, of a web member with a chord is called an *apex.* The distance from apex to apex on the same chord will be called a *panel length*, a panel being the part of the bridge so included.

The truss, Fig. 1, rests upon *abutments* at A and B and is unsupported at the distance or *space* AB.

The pressures exerted by the truss against the abutments are resisted by their *reactions* $V$, $V_1$, equal to them, on the principle that action and reaction are *ever equal.*

4. The following suppositions, only approximately realized in practice, will be made :

The reactions V, $V_1$, will be assumed to be vertical.

(Bow, in his "Economics of Construction," has given many illustrations of inclined reactions, due to friction at the abutments, resisting expansion or contraction of chords. Its influence is generally small when the end of the bridge rests upon rollers.)

It is assumed that the bridge members are jointed, or free to move, at the apices, and that the resultant resistance offered by each piece coincides in position and direction with the straight line connecting the joints or apices of that piece.

For the computation of the chords, main ties, braces and counters, the weight of bridge and load will be considered as concentrated at the apices of the chord that bears the roadway, the weight one-half panel either side of an apex *a*, on and over *ab* and *bc* being considered concentrated *at the* apex *a*.

Other suppositions will be noticed further on.

5. In Fig. 1, $w_1$, $w_2$ . . . ., are the panel weights on one truss due to the weight of the bridge or *dead load*, $w_6$, $w_7$, the panel weight due to live load at the corresponding apices.

Call the horizontal distances from $w_1$, $w_2$, . . . ., to B, $l_1$, $l_2$, . . ., respectively.

Now it is a law of Mechanics that *when any number of forces acting on a rigid body and in the same plane are in equilibrium, the algebraic sum of their moments about any point in the plane of the forces is zero.*

Take the point B as the *center of moments;* then since V acts upward and the weights $w_1$, $w_2$ . . . . downwards.

$$V \times \overline{AB} - (w_1 l_1 + w_2 l_2 + \ldots) + V_1 \times 0 = 0,$$

or denoting $(w_1 l_1 + w_2 l_2 + \ldots)$ by $\Sigma wl$, $\Sigma$ denoting *sum* of similar quantities, we have,

$$V = \frac{\Sigma wl}{\overline{AB}} \quad . \quad . \quad . \quad . \quad (1)$$

The above law of course holds if we

take moments about $a$, or any other point in the plane of the truss.

6. Again *it is a law of parallel forces in equilibrium that their algebraic sum is zero.*

$$\therefore V + V_1 - \Sigma w = 0 \quad . \quad . \quad (2)$$

$\Sigma w$ being put for $(w_1 + w_2 + \ldots)$.

When one reaction then is known the other can always be found.

The reactions V, $V_1$, with the weights of bridge and load $w_1$, $w_2$, . . . are called the *external forces.*

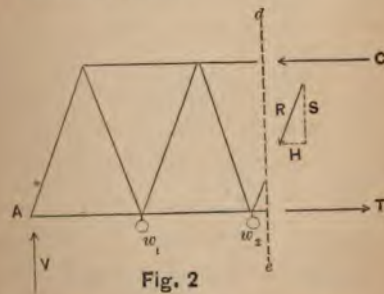



Fig. 2

7. Now suppose the truss cut along the line $\overline{de}$; conceive forces C, R, T, applied at the cut parts *equal* and *directly opposed to the resistances* of those men

bers, and let the part of the truss between B and $\overline{de}$ be removed. Then calling the sum of the weights $w_1, w_2 \ldots$ between A and $\overline{de}$, $\Sigma w$; the forces C, R, T, V, $\Sigma w$, must hold the part of the truss between A and $\overline{de}$ in equilibrium, since C, R and T are equivalent to the action of the external forces to the right of the section $\overline{de}$.

8. Denote the vertical component of R by S, its horizontal component by H. Call the forces V, $\Sigma w$, C, R, T, the *acting forces*. Then from Mechanics the algebraic sum of their vertical components equals zero

$$\therefore V - \Sigma w = S \quad . \quad . \quad . \quad (3)$$

Also the sum of the horizontal components of the acting forces equals zero,

$$\therefore C + H = T \quad . \quad . \quad . \quad (4)$$

S, the vertical component over any panel is called the *shearing force* for that panel and is always equal to the reaction V—, the sum of the downward forces from A to the section considered.

9. If $i$ denote the inclination of the

web member cut to the vertical then S sec. $i$ is the total stress on the web member.

From eq. (1), we find V; from eq. (3) S, whence the stress on any web member cut follows.

10. Note that if $S=V-\Sigma w$ is $+$, the *resistance* of web member cut acts in the same direction as V, *i.e.* upwards; R acts downwards, and the *strain on the tie-brace cut is compressive if its top leans away from the abutment* A; *otherwise tensile;* since in the first case H acts to the left, in the last it acts to the right in order that R, the resultant of H and S may act in the direction of the web member cut as was assumed. Let the reader conceive $\overline{de}$ removed one panel to the left and illustrate the last case with a drawing. Also the two following.

11. *When* $V-\Sigma w$ is $-$, then S acts upwards, therefore, *a web member whose top leans towards* A *is compressed, otherwise it sustains a tensile strain.*

The last two cases occur, for a uniform

load when the section $\overline{de}$ is taken to the right of the center.

These rules are especially useful in treating continuous girders or drawbridges.

12. *Maximum Strain on Web Members.*—The strain is greatest when the corresponding S is greatest; and S *is a maximum when the live load, the heaviest part in front, extends from the farthest abutment to the panel considered* ($\overline{de}$ Fig. 1).

(*a.*) For if any live load rests on the portion A*de* (Fig. 1), V is increased by a *part* of it only, whereas $\Sigma w$ is augmented by the whole of it, hence $S=V-\Sigma w$ is less than before.

(*b.*) Again, if any load on part B*de* is taken off, V is diminished, but $\Sigma w$ is the same as before, hence S is less than before.

(*c.*) If the live load, distributed as before, is placed with its front at the apex to the left of $\overline{de}$ and extending to the *nearest* abutment A, then $V_1<V$ and *since* $\Sigma w$ between B and $\overline{de}$ is greater

than $\Sigma w$ to left of $\overline{de}$ therefore, $S = V - \Sigma w$ is less than before.

(*d*.) The heaviest part of the live load must be in front, for then V is greatest.

We conclude as was enunciated.

13. In case (*c*) if the stress caused in a web member is of an opposite kind to that caused by the maximum shearing force, the member must be designed to resist alternately both stresses, or a counter must be added to the panel considered.

14. *Live Load.*—On this subject, see VAN NOSTRAND'S MAGAZINE for October, 1875, p. 305; also for May, 1877, p. 476; also the "Illustrated Albums" of many bridge companies.

The locomotive assumed for railroad bridges, in what follows, concentrates 84,000 lbs. on six drivers, three on each side, on a twelve feet wheel base. The locomotive and tender covers fifty feet of track; the thirty-eight feet not covered by the drivers before and behind the engine is supposed loaded with 2,000 lbs. per foot $\therefore$ total weight of locomo-

tive and tender is $84000 + 38 \times 2{,}000 = 160{,}000$ lbs.

15. *Computation of Floor Beams and Stringers.*—The floor beams extend from an apex of one truss to the corresponding apex of the other truss. The stringers resting on them lie under the rails or parallel to them.

Then for six feet panel lengths and under, the center drivers can concentrate $\frac{84000}{3} = 28000$ lbs. on floor beam or at center of stringers.

For twelve feet panels, let the center drivers rest on floor beam; the front and rear drivers being at center of adjacent panels, one-half their weight is supported by a *floor beam.* Its reaction from the $2000 \times 6$ lbs. in front and behind drivers $= 2\,\frac{2000 \times 6}{4} = 6000$; so that the floor beam sustains in all $28000 + 28000 + 6000 = 62000$ lbs. The stringers sustain 28000 at center, assuming that they are most strained when the *center drivers rest on their center.* For greater panels than twelve feet, assume approximately the 2000 lbs. per

foot, extending from front and rear drivers to the nearest floor beam, and reduce the load to an equivalent center load P for the stringers load. Thus for a panel length of twenty feet: Moment at center due to P is $\frac{1}{2}P\frac{20}{2}=5$ P. The moment at center due to the actual load is, 50000 $\times 10-28000 \times 6 - 8000 \times 8 = 268000$; which, equated with the other moment, gives P=53600.

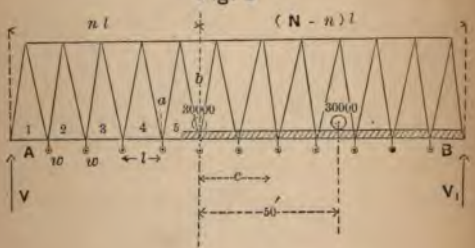
The maximum loads on floor beams are concentrated directly under the rails and are found by supposing center drivers to rest directly over floor beam. See arts. 43, 44 further on this subject. The following little table is made out on the above basis, and is intended to give average results:

| Length of Panel. | Floor Beams. | Stringers, equivalent center load. |
|---|---|---|
| 6 feet & under | 28000 | 28000 |
| 9 | 48666 | 28000 |
| 12 | 62000 | 28000 |
| 15 | 72400 | 38467 |
| $16\frac{2}{3}$ | 77492 | 44987 |
| 20 | 86800 | 53600 |

16. *Live Load for Web Members and Chords.*—The live load assumed for web members consists of two locomotives, as above, on 100 feet; there being not less than fifty feet between center driving wheels of locomotives; followed by cars weighing 2000 lbs. per foot for rest of span. For panel lengths over twelve feet, the disposition is as in Fig. 3, the 2000 lbs. per foot in front of drivers extending to middle of panel. For panel lengths less than twelve feet, the locomotive will be supposed to be without truck wheels in front and hence no weight is assumed before drivers. The locomotive excess over 2000 lbs. per foot is $84000-12\times2000=60000$ or 20000 lbs. on each pair of drivers; *hence we assume for web members the bridge loaded with* 2000 *lbs. per foot up to the middle of the panel considered, and a locomotive excess of* 60000 *lbs. at foremost apex, also* 60000 *lbs. fifty feet back of this;* or a greater *distance if the strains are thereby greater. For chord strains, we assume the bridge loaded with* 2000 *lbs. per foot*

*over the entire span, and with the locomotive excess, consisting of two weights of* 60000 *lbs. each not less than fifty feet apart; the latter to be so placed as to give maximum stresses on each chord panel in turn,* as will be fully shown in the sequel.

Fig. 3




17. If truck wheels are assumed in front of drivers, the shearing force is less than for the disposition above. For short spans especially, it seems desirable to assume as above that the foremost engine has no truck wheels, *i.e.*, when the panel lengths are less than 12 feet, or practically even for greater panel lengths.

18. *Web Strains.*—In Fig. 3,

$w$ = weight per panel of one truss with its share of roadway and cross bracing.

$p$ = weight per panel of cars[$=(1000\ l)$ pounds] for one truss.

$l$ = length of panel in feet.

E = locomotive excess ($=60000$ lbs.) for one truss.

$c$ = distance from front apex to its center of gravity (25 ft.)

N = number of panels (12 in Fig.)

$n$ = No. panel considered, numbered from A as in Fig.

$i$ = inclination of a tie or brace to the vertical.

Now to find the maximum shearing stress over the $n$th panel (5th in Fig.) by art. 16, the car load, 1000 lbs. per foot must extend to the middle of the $n$th panel; we also have, 30000 lbs. at apex on right of $n$th panel and 30000 lbs. 50′ to the right of the last. Next (art. 4), the *car load* $\frac{1}{2}$ panel either side of an

apex is regarded as concentrated at that apex.

19. Now take the right abutment as the center of moments. The lever arm of V is $Nl$. There are $(N-1)$ weights $w$, whose resultant acting at the center of the span, has a lever arm $=\frac{1}{2}Nl$. There are $(N-n)$ weights $p$, whose center of gravity is $\frac{1}{2}(N-n+1)l$ from right abutment ($\frac{1}{2}$ $\overline{Ba}$ in the Fig.) and lastly the locomotive excess E has a lever arm, $Nl-(nl+c)$

Therefore art. 5,

$$VNl=(N-1)w\tfrac{1}{2}Nl+(N-n)p(N-n+1)\frac{l}{2}+E[Nl-(nl+c)]$$

Or calling the shearing force over the $n$th panel $S_n$, we have, art. 7,

$$S_n=V-(n-1)w=\left\{(N-2n+1)\frac{w}{2}+(N-n)(N-n+1)\frac{p}{2N}+\frac{E}{N}(N-\frac{c}{l}-n)\right\} \quad (5)$$

When the rearmost engine is not on the bridge, E, in the preceding formula, *becomes* 30000 lbs. and c=o.

20. Having found maximum S over each panel, as *e.g.*, the 5th, the stress on the post = S sec. $i$, that on the tie is S sec. $i_1$; $i$ and $i_1$ denoting the respective inclinations of the post and tie to the vertical; art. 9. There is the same shearing stress on tie and brace over the same panel, and this evidently (see the reasoning of art. 7) holds when the posts are vertical as in the Pratt Truss, or the ties vertical as in the Howe Truss, or when the ties and braces are equally inclined as in the Warren Girder, or unequally inclined as in the above figure. We shall use this equation in discussing the bowstring and other forms of girder likewise. An example will best illustrate the use of the equation and the theory of counters.

21. *Example.*—Let the span AB=200 feet, divided into N=12 panel lengths of 16′ 8″ each; weight of bridge 336000 lbs. or 168,000 lbs. to each truss $\therefore w=\frac{168000}{12}=14000$; live load as in art. 16 $\therefore p=1000$ $l=\frac{50000}{3}$, since $l=\frac{50}{3}$; E=60000, $c=25$. Substituting these values in formula (5), we get:

$$S_n = (13-2n)\,7000 + (12-n)(13-n)\,694 + (10\tfrac{1}{2}-n)\,5000.$$

| | S | $\Delta_1$ | $\Delta_2$ |
|---|---|---|---|
| $S_1 = 77000 + 11\times12\times694 + 47500 =$ | 216108 | 34268 | — |
| $S_2 = 63000 + 10\times11\times694 + 42500 =$ | 181840 | 32880 | 1388 |
| $S_3 = 49000 + 9\times10\times694 + 37500 =$ | 148960 | 31492 | 1388 |
| $S_4 = 35000 + 8\times9\times694 + 32500 =$ | 117468 | 30104 | 1388 |
| $S_5 = 21000 + 7\times8\times694 + 27500 =$ | 87364 | 28716 | 1388 |
| $S_6 = 7000 + 6\times7\times694 + 22500 =$ | 58648 | 27328 | 1388 |
| $S_7 = -7000 + 5\times6\times694 + 17500 =$ | 31320 | 25940 | 1388 |
| $S_8 = -21000 + 4\times5\times694 + 12500 =$ | 5380 | 24552 | 1388 |
| $S_9 = -35000 + 3\times4\times694 + 7500 =$ | −19172 | — | — |
| $S_{10} = -49000 + 2\times3\times694 + 5000 =$ | −39836 | — | — |
| $S_{11} = -63000 + 1\times2\times694 + 2500 =$ | −59112 | — | — |
| $S_{12} = -77000 \qquad\qquad =$ | −77000 | — | — |

The rearmost 30000 locomotive excess leaves the truss for $S_9$, whence the formula is then modified by putting $c=o$ and $E=30000$, to compute $S_9$, $S_{10}$ and $S_{11}$.

22. The common differences for the terms $(13-2n)$ 7000 and $(10\frac{1}{2}-n)$ 5000 are $2\times 7000$ and 5000 respectively; from which those terms are quickly computed. Column $\Delta_1$ is found by subtracting each value in column S from the preceding value. Column $\Delta_2$ is the common difference of the quantities in column $\Delta_1$. In fact if in the preceding equation we change $n$ to $n+1$ and subtract the last equation from the first we get,

$$(S_n - S_{n+1}) = -\frac{p}{N}n + \left(w + p + \frac{E}{N}\right),$$

the equation of a straight line which makes an angle with the axis of abscissas $(n)$ whose tangent is $-\frac{p}{N}$.

Giving values to $n$ : 1, 2, 3 . . . . , the common difference between successive values is, $\Delta_2 = \frac{p}{N}$. On computing $S_1$ $\Delta_1$

and $\Delta_2$; by reversing the above method of deducing columns $\Delta_1$ and $\Delta_2$ we can find the various values of S.

23. The "shears" may be found graphically, if desired, by drawing the straight line given by the equation above, taking off in dividers the differences between successive shears ($S_n - S_{n+1}$) which are represented by the ordinates to the line, and subtracting these first differences in order from the line taken to represent $S_1$, thus giving lines which measured to a scale will give $S_2$, $S_3$ . . . , provided always that both engines remain on the bridge.

Thus making $n=1$, we have

$$(S_1 - S_2) = -\frac{p}{N} + w + p + \frac{E}{N},$$

for the *difference* between the shears in the first and second panels. Lay this difference off vertically above the lower apex one panel to the right of the left abutment, regarding the lower chord as the axis of abscissas ($n$). Also lay off the value ($S_6 - S_7$), say, at the 6th apex from the left abutment; the line joining

the extremities of these ordinates will cut off the successive differences $\Delta_1$ from ordinates erected at apexes 2, 3, 4, . . . . . These ordinates, can be laid off successively on the line equal to $S_1$ by scale; thus giving $S_2$, $S_3$ . . . . , as before mentioned.

24. Having thus found $S_1$, $S_2$, . . . , the strains in the web members over panels 1, 2, . . . . are $S_1$ sec. *i*, $S_2$ sec. *i*, . . . . When a web member over the *n*th panel becomes vertical, its mass strain is simply $S_n$. If we form a column, S sec. *i*, to the right of column S (art. 22), and deduce $\Delta_1$ and $\Delta_2$ *from it*, we detect any error that may occur in multiplying by sec. *i*.

By art. 10, *when* $S_n$ is +, the web members whose tops lean away from A act as struts; those whose tops lean toward A as ties. In this case, S is + for the first 8 panels, or 2 panels past the center. Now if the live load is supposed to move on from A toward B, we prove similarly that the web members whose tops lean $\left\{\begin{matrix}\text{away from}\\ \text{towards}\end{matrix}\right\}$ B act as

$\left\{ \begin{matrix} \text{struts} \\ \text{ties} \end{matrix} \right\}$. Therefore each web member, in this case, in panels 5, 6, 7 and 8, suffers both compression and tension in turn and must be designed for the maxima of both strains, *i.e.*, counter braced. Thus max. S over the 5th panel is when the live load extends from the farthest abutment, and equals $S_5 = 87364$. But when the live load extends from the nearest abutment to the 5th panel *from that abutment* as from B to panel marked 8 in Fig. 3, $S = S_8 = 5380$; and, as just shown, the web members previously designed as $\left\{ \begin{matrix} \text{struts} \\ \text{ties} \end{matrix} \right\}$ for S = 87364, must now be designed as $\left\{ \begin{matrix} \text{ties} \\ \text{struts} \end{matrix} \right\}$ for S = 5380, also. Similarly the web members of panel 6 are designed as $\left\{ \begin{matrix} \text{struts} \\ \text{ties} \end{matrix} \right\}$ for a stress of 58648 *sec. i*, and also as $\left\{ \begin{matrix} \text{ties} \\ \text{struts} \end{matrix} \right\}$ for a maximum stress of 31320 *sec. i*. We then design the web members up to the middle of the truss for the max. stresses, and those panels past the center

for which S is positive, which may be numbered now from the other abutment, if preferred, (after S is found by form 5 by its numeration), have their web members designed for the lesser stresses of an opposite character to the maximum stresses.

25. If preferred, in place of causing the same piece to act *both* as a strut and a tie, we may insert a counter in the panel to bear one of the strains, designing the main ties or braces of that panel so that they cannot take a reverse strain and the inserted member is thus compelled to take it.

Thus in Figs. 5 and 6 the dotted lines are *counters that bear but one kind of strain*, like the main ties and braces of the truss.

26. When the live load, engines in front, extends from the *farthest* abutment, $S_n$ is a maximum for the $n$th panel by art. 12.

When the live load, engines in front, extends from the *nearest* abutment to the $n$th *panel, S being* +, the strains induced

in the web members of the $n$th panel are a maximum of an opposite character to the first.

The proof is the same as in cases *a*, *b* and *d* of art. 12.

27. *When the load extends from the nearest abutment and* $S_n$ (*of eq.* 5 *is* $-$, as in panels 9, 10, 11, 12 of bridge assumed, the strains are not reversed (see art. 10), but we find from eq. (5) the *minimum strains* that can ever come on the web of the panels considered.

Thus we see in the example, art. 21, that $S_9$, $S_{10}$, $S_{11}$, are less numerically than if there is no live load on bridge, for the term $(N-2n+1)\frac{w}{2}$ involving the dead load is $-$ when $n > \frac{1}{2}N$, whereas the two terms involving the effects of the live load are always positive.

Then, reasoning as in art. 12, we see that the *positive terms* are less for any live load in front of panel considered, or for any live load taken off behind the panel, and that the locomotives must be in front, therefore $S_n$ is least numerically

when the load, engines in front, extends from the nearest abutment, $S_n$ *being negative.*

28. Observe that for a dead load alone, $p=o$, $E=o$, that $S_n = \frac{1}{2}(N-2n+1)\,w$, which gives the same value numerically, but with a different sign, whether $n=\frac{1}{2}N+m+1$, or $n=\frac{1}{2}N-m$, or for panels equally distant from the center. Thus the web members equally distant from the center and similarly placed with respect to it, are equally strained by a uniform load.

29. It is seen by reference to the method of deducing eq. (5) that it gives the maximum shearing force at any panel for any girder, framed or not, of span AB, *numbered and loaded as* AB *is in Fig.* 3; hence it applies to the inclined members of the Pratt, Howe, or triangular trusses (Figs. 5, 6 and 7) whether the load (live and dead) is all supposed to rest on the lower or upper chords, or both, provided the panel members begin *at* the abutment as in Fig. 3.

It is *well to note* carefully the position

of the front engine that gives maximum strains on the *vertical* members of the Pratt or Howe types. Thus, in Fig. 3, the *shear* is the same on the two inclined web pieces of a particular panel. Now conceive the STRUTS TO BECOME VERTICAL by moving their tops forward; the *max.* shear they ever sustain is the same as that of the tie reaching to their top from the front engine, for the *through bridge;* but for a *deck bridge* this is not so. The max. strain on a vertical post then obtains when the front engine is directly over the post; whilst the ties are most strained when the engine is directly over the post that connects with their lower ends (art. 12). For a Howe bridge, THE VERTICAL TIES are most strained when the engine is at their feet for *a through bridge*, or at the top of the brace that connects with their feet for *a deck bridge.* The live load must never extend so far that part of it must be subtracted from V in finding S (art. 12).

Similarly, the *minimum shear* a web piece ever bears (art. 27) is when the

live load extends from the nearest abutment as far as may be without $\Sigma w$ being increased by any of the live load in the expression, $S = V - \Sigma w$ (art. 7).

30. The formula does not apply to the Warren girder, or to Fig. 3, when the load is on the upper chord (concentrated at the apices); since the weights are not then distributed as in Fig. 3.

The methods of arts. 5 and 7 can then be used.

31. Let us now ascertain the extent of the error made by assuming that the load $\frac{1}{2}$ panel length either side of an apex is supposed concentrated at that apex.

*a.* Thus in Fig. 3 we virtually consider the $\frac{1}{2}$ panel of live load next B as removed. Except for very short spans its influence is very slight. In this case V, and therefore S, would only be increased by it $\left(\frac{p}{2}\,\frac{l}{4}\right) \div 12\ l = 174$ pounds.

*b.* We have also disregarded the dead load $\frac{1}{2}$ panel next A and B. Including it, V is increased by $\frac{1}{2}\ w$; but $S_n = V$

$-\Sigma w$ is the same as before since $\Sigma w$ is likewise increased by $\frac{1}{2}\ w$.

*c*. If the weight of web members is not supposed concentrated at the apices, but distributed as it really is, then $S_n = V - \Sigma w$ diminishes in the same panel the further the section taken (Fig. 2) is from the abutment; and still more if any chord piece and load be supposed borne (as it really is) at the apices of that chord piece.

This case represents exactly the true solution. Thus in $S = V - \Sigma w$, the term $\Sigma w$ equals the weights of chords and loads borne at apices from A to section taken, + the weight of web to section. Such refinement is generally unnecessary for medium spans.

In the triangular truss, shown in Fig. 7, the loads are supposed borne at the apices of either chord alternately so that one source of error is eliminated for this truss. In the Pratt the posts bear one panel weight of upper chord + part of their own weight above section taken over that given by eq. (5), for a *through*

bridge; whilst for a deck bridge eq. (5) gives an excess of one panel of lower chord and weight of post above section over the true strain. A figure will illustrate this; also the modification for the ties of the Howe Truss.

*d.* In Fig. 3, we have supposed also the live load on the *n*th, or 5th panel in the Fig., extending up to the middle of the panel, to be concentrated at the right apex. Actually *part* of it is conveyed by the stringers and floor beams to *a* directly. Call P the reaction at *a* due to this part. Now V will be larger than on the former supposition and will be augmented by a *part* of P whereas $\Sigma w$ is increased by the whole of P; therefore $S=V-\Sigma w$ is less than given by eq. (5). Eq. (5) is then on the side of safety.

32. The true value of S can be readily found, but it is not advisable in practice to enter into such refinements, for the supposition of hinged joints, &c. (art. 4) is never exactly realized in practice; hence the actual strains in a structure

probably always differ from the computed, especially the components acting on the fibers most strained; again the hurtful effects of vibration, oscillation and impact, modify in an unknown manner the strains due to a statical load, therefore it seems useless to insist upon strict accuracy in such statical calculations.

33. To test further the method of apex loads: Suppose a uniform load $q$ per foot, to extend from the apex $b$, to the right of $a$, a distance $x$ to the left from $b$. Call $a'$ and $b'$ the parts of this load borne at the apexes $a$ and $b$; then we can write the reaction $v$ at A, due to $a'$ and $b'$,

$$v = ma' + nb',$$

$m$ and $n$ being certain *proper* fractions.

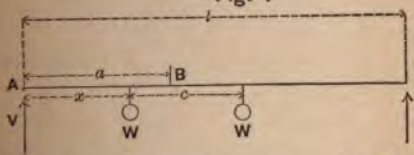
The value of S over the 5th panel, due to the above load, is then, $S_5 = V - a' = nb' - (1-m)a'$, which is *less* than $nb'$. Now when $x \leqq l$, $b' \leqq \frac{1}{2}ql$ $\therefore S_5 < nb' \leqq n\frac{1}{2}ql$. But if we suppose (as in art. 18) that the load extends to the middle of the panel, and that the whole of it, $\frac{1}{2}ql$, is

concentrated at $b$; $S_b$ would be, $n\frac{1}{2}ql$, which is thus always greater than the *actual* shearing force, which we found above to be less than, $n\frac{1}{2}ql$, whether the uniform load covered the whole or a part of the panel $ab$. The supposition is then on the side of safety.

34. *Chord Strains.*—The live load assumed has been given in art. 16. Let us first ascertain how two weights, each: W=60000 lbs. and $c$=50 feet apart, are to be placed so as to give maximum strains on any chord panel. Assuming the notation in Fig. 4, we have,

$$V=\frac{2\,W\,(l-x-\frac{1}{2}c)}{l}$$

Fig. 4




whence the moment at any point B, *between* the two weights is,

$$M=Va-W(a-x)$$
$$=W\left\{\left(a-\frac{ca}{l}\right)+\left(1-\frac{2a}{l}\right)x\right\}$$

Now if the two engines can get on the truss $\frac{c}{l}<1$ $\therefore \left(a-\frac{ca}{l}\right)$ is positive. We must suppose $a<\frac{1}{2}l$, for one-half of the truss will be subjected to the same maximum strains as the other half, hence we need only consider one-half. It follows that $\left(1-\frac{2a}{l}\right)>0$, hence M increases with $x$; so that for $x=a$, *or when the front engine is at the section* B, M *is a maximum for that section.* When $x>a$, V is less than before, and hence $M=V'a$ is less than for the maximum just found.

Fig. 5

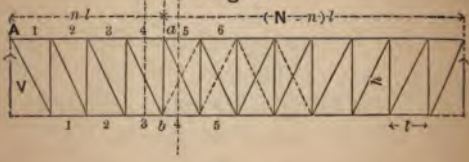



Fig. 6

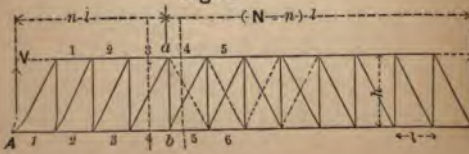
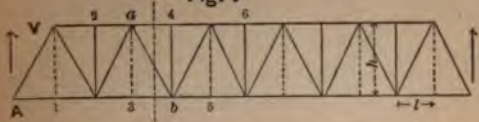


Fig. 7




35. *For a truss, one weight* W *is on the same vertical with the apex taken as the center of moments,* corresponding to B of Fig. 4, *the other weight,* $c=50$ *feet, from it on the side of the farthest abutment.*

For *center chord panels,* $a=\frac{1}{2}l$ $\therefore$ $M=\left(a-\frac{ca}{l}\right)W$; which is independent of $x$; so that the weights, 50 feet apart, can be placed in any position provided they are not both on one side of the center panel.

36. Let N=number of panels (12 in Figs. 5, 6 and 7).

$h$=height of truss, center to center of chords.

P=uniform load per panel $=w+p$ (art. 18).

E, $c$, $l$, as defined in art. 18.

$t_n$ = strain on $n$th panel of lower chord,

$c_n$ = strain on $n$th panel of upper chord,

*the chords being numbered as in Figs.* 5, 6 *and* 7.

It is immaterial whether the loads be considered as concentrated at upper or lower apices or both; hence the results are true whether the trusses are "through" or "deck." First consider the effect of the uniform load alone. For maximum chord strains, the car load must cover the whole truss, since any part of it causes an *upward* moment, giving compression in the upper chord, and tension in the lower one.

Conceive the truss cut in two, as per dotted line, through the panels marked 4 (Figs. 5, 6 and 7) of upper chord, and the right part of the truss removed, and apply as in art. 7 forces equal and opposed to the *resistances* of the cut pieces of the left part. The algebraic sum of the moments of these forces, V, and the loads *about any point* must be zero (art. 5). Suppose the counters (dotted *lines Figs.* 5 and 6) removed, if any

should be cut, and *take the center of moments at the intersection of the web member and either chord* to find the strain on the panel of the other chord piece cut. The moment of the web member and chord passing through center of moments is thus zero. This is a general method applicable to any structure and conduces to simplicity.

Thus if the $n$th (=4th in the Figures) panel of the upper chord is cut, take $b$ as the *center of moments*, $b$ being the intersection of the web member cut with lower chord. Then $c_n h$=moment of $c_n$; $Vnl$=moment of V, and $(n-1)P\frac{1}{2}nl$= moment of the $(n-1)$ apex loads, P between A and $b$, the lever arm of their resultant being $\frac{1}{2}nl$, since they are symmetrically disposed with respect to a point half way between A and $b$. We have then (art. 5), since $V=(N-I)\frac{P}{2}$,

$$c_n h = Vnl-(n-1)P\tfrac{1}{2}nl$$

$$=[(N-1)n-(n-1)n]\frac{Pl}{2}$$

$$\therefore\ c_n=\frac{(N-n)n}{2h}Pl$$

By giving any value to $n$, we find $c_n$ corresponding.

37. Next, suppose section taken across panel $4=n$ say of lower chord (Figs. 5 and 6); the center of moment is then at $a$, vertically above $b$; therefore as above we find

$$t_n = \frac{(N-n)n}{2h} Pl$$

the same value as for panel $n$ of upper chord, as it should be; since V, and the loads P, have the same lever arms as before.

The same formulæ apply to Fig. 7; only $n$ must have successively the values 1, 3, 5 . . . for the lower chord and 2, 4, 6 . . . for the upper, since the center of moments for any chord panel is at the apex opposite on the other chord, thus giving a uniform strain on two chord panel lengths in turn, as marked in Fig. 7.

38. *Locomotive Excess.*—By art. 35 we suppose $\frac{E}{2}$ placed at $a$ or $b$ (Figs. 5

and 6) and $\frac{E}{2}$, $50=c$ feet to right, to get the chord strains on panels four upper and lower chords, since $a$ or $b$ is the center of moments corresponding to panels four as marked. The distance from $a$ or $b$ to A being $nl$, the *reaction* at A of E is (art. 19), $\frac{E}{Nl}[Nl-(nl+c)]$; its lever arm about $a$ or $b$ is $nl$, so that the moment on the $n$th panel of upper or lower chord as marked, due to E is,

$$M=\frac{E}{Nl}[Nl-(nl+c)]nl=\frac{E}{N}\left(N-\frac{c}{l}-n\right)nl$$

For Fig. 7 the same formula holds, as is easily seen, taking care to give $n$ the values marked on the chords in turn. Thus $\frac{E}{2}$ is at $b$ to find max. $c_4$ and at $a$ to find max. $t_3$. The value of $c_n$ or $t_n$ due to E is therefore

$$\frac{E}{N}\left(N-\frac{c}{l}-n\right)\frac{nl}{h}$$

39. Combining this with the value

previously found due to the uniform load we have as the maximum strain that can come on a chord panel, $n$, as numbered in Figs. 5, 6 and 7

$$c_n = t_n = \left(\frac{Pl}{2h}(N - n) + \frac{El}{Nh}(N - \frac{c}{l} - n)\right)n \quad . \quad (6)$$

40. *Example.*—As in art. 21, let span $=200$ feet, $N=12$, $l=\frac{50}{3}$, $P=w+p=14000+16666=30666$, $E=60000$ lbs., $\frac{c}{l}=1\frac{1}{2}$ and assume the height of truss at twenty-eight feet.

Eq. (6) becomes then

$$c_n = t_n = [9127(12-n) + 2976(10\tfrac{1}{2}-n)]n$$

By making $n$ successively, 1, 2, 3, 4, 5, 6, we form the following table: The second differences are constant, as before, thus checking the work.

| | $t=c$ | $\Delta_1$ | $\Delta_2$ |
|---|---|---|---|
| $c_1=t_1=(100397+28272)1=$ | 128669 | 104463 | — |
| $c_2$ " $t_2$ " ( 91270+25296)2" | 233132 | 80257 | 24206 |
| $c_3$ " $t_3$ " ( 82143+22320)3" | 313389 | 56051 | 24206 |
| $c_4$ " $t_4$ " ( 73016+19344)4" | 369440 | 31845 | 24206 |
| $c_5$ " $t_5$ " ( 63889+16368)5" | 401285 | 7639 | 24206 |
| $c_6$ " $t_6$ " ( 54762+13392)6" | 408924 | — | — |

41. The first and second terms in the ( ) are computed by the common differences, 9127 and 2976.

Thus, in a few minutes time, the chords are accurately calculated for their maximum strains. The strains on the triangular truss Fig. 7 are, as before explained, for the upper chord $c_2$, $c_4$ and $c_6$ respectively; for the lower chord $t_1$, $t_3$, $t_5$. In Fig. 5 the greatest strain on any lower chord panel is, $t_5$. In Fig. 6 the greatest strain on any upper chord panel is $c_5$; the strains on the other half of the truss being similar to those of the first half.

The strains thus far found may be marked on larger drawings than those given on the corresponding parts. Let us tabulate the results thus far found in the following table *for truss, Fig.* 7. The length of a diagonal $= \sqrt{28^2 + (16\frac{2}{3})^2} =$ 32.6; and *sec.* $i = \frac{32.6}{28} = 1.165$. Multiplying the values $S_1$, $S_2$ . . . art. 21, by 3 get the strains on the web

members given in the column marked "Strain"; the *numeration* for the web members being the same as on Fig. 3, as given for the corresponding shearing forces. Thus $S_1$ *sec.* $i$=strain on end brace, $S_2$ *sec.* $i$ for the next *brace* over panel 3, $S_5$ *sec.* $i$ for next brace over panel 5, $S_7$ *sec.* $i$=strain on tie over sixth panel when it acts as a brace, $S_8$ *sec.* $i$=strain on brace over fifth panel when it acts as a tie, (see art. 24). The chord strains are designated as in art. 36, Fig. 7.

Column ($d$) gives the outer diameter in inches of the *compression member*; column $\left(\frac{l}{d}\right)$, the ratio of its length to its diameter; column ($th$), the thickness of metal in inches; column ($\theta$), the ratio of the *least* strain that can ever come upon a member to the *greatest* strain that can ever come on the member; column ($b$), the strain for square inch allowed.

The columns headed "Area," "Length," "No." give respectively, the area of the cross section of the member in square

inches, its length and the number of pieces similarly strained in two trusses.

Column ($k$) gives the weight of wrought iron of section one square inch, and one foot long$=\frac{10}{3}$ pounds.

The next column gives the "weight" of member or members in pounds, found by multiplying together the four previous columns.

The last column is a summary, giving the weights on computed strains, in order, of braces and posts, upper chord, main and counter ties, and lower chord, on two trusses.

Column ($b$) will be explained further on.

42. Given the "strain," and $b$=safe strain per square inch allowed, $\frac{\text{strain}}{b}$ =area, which is put in "area" column. The least section of a post allowed was nine inches, for the vertical posts, that simply sustain one panel of upper chord and bracing. The counter brace was supposed a latticed member. Its total area is that due to its acting both as a

| Piece. | $a$ | $\frac{a}{d}$ | th. | Strain. | $U$ | $\delta$ | Area. | Length | No. | $k$ | Weight. | Totals. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ″ | | ″ | | | | | | | | lbs. | |
| Brace 1 | $13\frac{1}{16}$ | 30 | $\frac{21}{16}$ | 251766 | .36 | 5340 | 47.15 | 32.6 | 4 | $\frac{10}{8}$ | 20495 | |
| 3 | ″ | ″ | $\frac{16}{16}$ | 173538 | .27 | 4990 | 34.8 | ″ | ″ | ″ | 15126 | |
| 5 | ″ | ″ | $\frac{11}{16}$ | 101779 | 0 | 3930 | 25.9 | ″ | ″ | ″ | 11258 | |
| *Counter 7 | ″ | ″ | $\frac{1}{2}$ | 36487 | 0 | 3440 | 10.6 | ″ | ″ | ″ | 4607 | |
| Laticing & Angles | | | | | | | | | | | 4000 | |
| Vertical Posts | 6 | 56 | $\frac{1}{3}$ | 2500 | 1. | 280 | 9. | 28. | 10 | ″ | 8400 | 63886 |
| Upper Chord $c_2$ | $13\frac{1}{8}$ | 15 | $\frac{2}{3}$ | 233132 | .39 | 9050 | 25.76 | $\frac{10}{8}$ | 4 | ″ | 11450 | |
| $c_4$ | ″ | ″ | 1 | 369440 | ″ | 9270 | 39.85 | $\frac{10}{8}$ | ″ | ″ | 17711 | |
| $c_6$ | ″ | ″ | $\frac{7}{6}$ | 408924 | ″ | 9270 | 44.11 | $\frac{10}{6}$ | ″ | ″ | 9802 | 38963 |
| Main Tie 2 | | | | 211844 | .32 | 9900 | 21.4 | 32.6 | ″ | ″ | 9300 | |
| 4 | | | | 136850 | .16 | 8700 | 15.73 | 32.6 | ″ | ″ | 6836 | |
| 6 | | | | 68325 | 0 | 7500 | 9.11 | 32.6 | ″ | ″ | 3960 | |
| Counter 8 | | | | 6268 | 0 | 7500 | 2. | 32.6 | ″ | ″ | 880 | |
| Suspenders | | | | 45000 | .13 | 8470 | 5.3 | 28. | 12 | ″ | 5936 | 26912 |
| Lower Chord 1 | | | | 128669 | .39 | 10420 | 12.35 | $\frac{10}{8}$ | 4 | ″ | 5488 | |
| 3 | | | | 313389 | ″ | 10420 | 30.1 | ″ | ″ | ″ | 13376 | |
| 5 | | | | 401285 | ″ | 10420 | 38.51 | ″ | ″ | ″ | 17120 | 35984 |

* Total area tie 6 and counter 7=9.1+10.6=19.7, requiring for a rectangular cross section 8 plates, $13\frac{1}{16}\times\frac{1}{2}$. Under side half latticed bars, $2''\times\frac{3}{8}''$. Angle irons at 4 corners, $2\frac{1}{2}''\times2\frac{1}{2}''\times\frac{3}{8}''$.

strut and main tie (6). The counter tie (8) is supposed enclosed in brace (5). The main braces and upper chords were assumed to be "Phœnix columns."

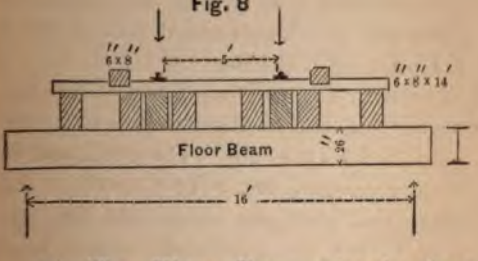
43. Fig. 8 is a section of flooring. The rails, spikes, chairs, &c., are assumed to weigh 42 lbs. per foot. Assume the weight of a cubic foot of white pine timber at 36 lbs.; and the cross ties $1\frac{1}{2}$ feet from center to center. Then the weight of cross ties and guard timbers (placed parallel to and on the outside of the rails) per longitudinal foot is $6\times 8$ $(14+3)\frac{2}{3}\cdot\frac{36}{144}=124$ lbs.

In art. 15, we found the maximum center live load on the *stringers* of a $16\frac{2}{3}'$ panel to be 44987 lbs. The dead load of rails, cross-ties, &c.$=(42+124)\frac{50}{3}$. Assume that stringers weigh 325 lbs. per foot. Then the equivalent center load of one panel of rails, &c., and stringers is, $\frac{1}{2}491\times\frac{50}{3}=4092$, making the total center load on stringer, 49080 lbs. If we add 50 per cent. to this to allow for impact, &c., and take 1000 lbs. as the safe *strain for pine*, we find that we must

have 6 stringers under the rails of $9\frac{1}{8}'' \times 20''$ cross section. The 8 stringers will thus weigh 365 lbs. per foot of rail.

*Four Iron stringers* will weigh 300 lbs. per foot (two under the rails) if their depth is 26 inches; neglecting the influence of the web $\frac{3}{8}''$ thick, which is about equivalent to the loss in the rivet holes in the tension flange. The method of computation is the same as for the floor beam.

Fig. 8




44. The *Floor Beam*, also the floor beam loops sustain a max. live load of 77492 lbs. (art. 15). To this add say 3000 lbs., weight of floor beam, and $531 \times \frac{5.0}{3} = 8850$ lbs. for stringers, rails, &c.; *giving a total* load, say on the rails

over the floor beam of (77492+11850) =89342 lbs., or 44671 lbs. on each rail. The moment at the center is, 44671 (8—2½) = 245690 *foot lbs.* = 2,948,280 *inch lbs.*, which must equal the resisting moment, $fda$=7500×26×15.1, of the I section ($f$=safe strain=7500 lbs. per square inch, $d$=26=total depth and $a$= area of one flange=15.1 square inch). The *cross section*, assuming a thickness of web of $\frac{1}{3}''$, the depth between flanges being 24 inches, is 30.2+8=38.2 square inch; giving the weight of one floor beam=38.2×17½×$\frac{10}{3}$=2228 lbs. Therefore 11 floor beams weigh 24500 lbs. The floor beam loops are put at 5000 lbs.

For such depths of beams (26″), it is advisable to diminish the depth of girder from near the center towards the points of support (see Boller's "Iron Highway Bridges," p. 64), both for economy of beams and loops as well as for appearance sake. The saving so effected will be assumed approximately equal to weight of rivets and stiffeners; which is sufficiently correct for the purposes of *these estimates*.

45. We are now enabled to find the maximum load on the suspenders of Fig. 7; thus,

| | |
|---|---|
| Live load on one floor beam.... | 77492 lbs. |
| Dead load of floor beam..... | 2228 " |
| Stringers, rails, &c. $(300+166)\frac{12}{8}$ = | 7770 " |
| One panel lower chord, &c..... | 2000 " |
| | 89490 " |

Or, say, 45,000 pounds borne by the suspenders of one panel of one truss.

46. Assuming a wind surface, when the bridge is covered with cars, 16′ high $\times$ 200′ long; the intensity of the wind being taken at 30 pounds per square foot, the uniform horizontal pressure per panel is $16 \times \frac{50}{3} \times 30 = 8000$ pounds $= w$. The trusses are connected between the chords by bracing similar to that of the Pratt truss, Fig. 5 at the center; hence, the shearing stress, occasioned in this transverse bracing by the wind pressure is given by Eq. (5), on making $p$ and E zero, and $w = 8000$. The strain on any member then, is,

$$S_n \sec. i = \tfrac{1}{2}(N - 2n + 1)\, w \sec. i$$
$$= 4000\ (13 - 2n) \sec. i.$$

Allowing for tension 1500, and for compression 5000, pounds per square inch, the rods will average two or three square inches cross section; and their total weight, including bolts, nuts, etc., is put at 5400 pounds. The cross struts and portals are assumed to weigh 6000 pounds.

47. It will suffice, for our purposes, to add twenty per cent. to the computed material in upper chord and posts for castings, etc.; and fifteen per cent. to weight of ties and lower chord for bolts, nuts, eyes and pins; which allowances I find given by Mr. C. Shaler Smith in his "Comparative Analysis of the Fink, Murphy, Bolman and Triangular Trusses" Baltimore, 1870.

From the foregoing data we form the following :

BILL OF MATERIALS.

*Triangular Truss*—200′ *span*—28′ *high.*

| | | |
|---|---|---|
| Braces and Posts | 63886 | lbs. |
| Upper Chord | 38963 | " |
| 20 per cent. on two last | 20570 | " |
| Main Ties and suspenders | 26912 | " |
| Lower chord | 35984 | " |
| 15 per cent. on two last | 9434 | " |
| Floor beam loops | 5000 | " |
| Lateral Bracing | 11400 | " |
| 11 Iron floor beams | 24500 | " |
| Iron Stringers | 60000 | " |
| Rails and Cross Ties | 33200 | " |
| Total weight | 329849 | " |
| Assumed weight | 336000 | " |
| Assumed weight too great by | 6151 | " |

The bridge weight assumed, 336,000 pounds, is, consequently, too great by 6151 pounds.

49. The above allowances for castings, connections, etc., are intended as averages common to several trusses that will be examined. These details are varied indefinitely by builders. All the steps have been given, however, to render adaptation to any particular design easy.

50. In the table, art. 42, we assumed "$d$"$=13\frac{1}{3}''$ for upper chord. If we put $d=12$ for upper chord and braces, the total weight of bridge is found to be 6340 lbs. greater than before. If we assume that the increased weight of castings, rollers, pins, &c., is not over 2000 to 3000 lbs., there is of course economy in employing the greater diameter; and it may be found economical to increase it still further; taking care that a proper thickness of metal is maintained, say not less than $\frac{1}{8}$ inch.

It is hardly necessary to remark that from the "area" and "$d$" columns, we can find the inner diameter $d_1$ and hence the thickness of metal. Thus $\frac{\pi}{4}(d^2-d_1^2)$ $=$"area," from which $d_1$ is obtained and $\frac{d-d_1}{2}=$ thickness of metal.

51. The weights just found are in excess of average practice. This is partly because he assumed an engine weighing 84,000 lbs. on drivers in space of twelve feet, the total weight of engine and

ender, covering fifty feet, being taken at 160000 lbs., whereas a common specification gives the live load as 60000 lbs. on twelve feet, the engine and tender weighing 130000 lbs. The car loads ordinarily assumed are from 2000 to 2240 lbs. per foot. We have also used smaller unit strains than usual for some bridge members. Since the Ashtabula accident it was proposed to the Ohio Legislature to assume for bridge computations a live load of two locomotives and tenders, the locomotives weighing 91200 lbs. on 12½ wheel base, followed by cars weighing 2250 lbs. per foot of track.

In the Keystone Bridge Company's "Album" p. 22, we read; "For main lines of traffic, it is not considered prudent to assume less than 40 tons in a span of twelve feet,—stringers spanning over 12 feet should be sufficiently strong to carry 1½ tons per foot for each additional foot." (The 2000 lbs. ton is meant.) Considering the fact that engines are built with us weiging 100000 lbs. on 19 feet, it would seem that, for roads that

use such engines, the live load assumed, art. 14, is certainly not too great. For secondary lines a less weight might be assumed, if the road is to continue secondary.

Mr. C. Graham Smith, in a paper read before the Liverpool Enginering Society, of which he was President, June 20, 1877, and republished in the "Engineering News," Chicago, says:

"Mr. Benjamin Baker has conclusively proved, in his admirable little work on 'Long Span Railway Bridges' that there are many circumstances, such as badly maintained permanent way, inclined cylinders, and unbalanced portions of the mechanism of locomotives, together with great weight and length of engines, combined with short wheel base, which will at times render the effective load on one axle equivalent to thirty tons . . . .

"With shallow cross girders, oscillations are set up by heavy continuous traffic which will soon shake loose rivets and bolts and perhaps the connections with the main girders . . . .

"Here is an actual example, recorded in the before mentioned 'Long Span Railway Bridges.' The platform of the railway bridge over the Regent's Canal was constructed, owing to local circumstances, with cross girders only

8 inches deep and 14 feet 6 inches span. With a view of compensating as much as possible for want of depth, longitudinal stiffening girders 18 inches deep were placed at a distance of 2 feet 3 inches from the outer edge of each rail; each cross girder was also well secured by tee iron and gusset plates to the main girders. The bridge, notwithstanding that with 15 tons to one axle, it was so designed that the iron should not be strained more than 4 tons per square inch, completely gave away in four years. Mr. Baker attributes the failure to the employment of a 45 ton engine, the wheel base of which was 14 feet; the ends consequently overhung very much, which would greatly assist in producing oscillations and other undesirable consequences."

Similar facts have been recorded in this country, though the use of trucks causes our locomotives to run much steadier than English ones. The use of deep girders is then advisable, and 50 per cent. may be added to the live load of stringers as an additional precaution. Experience, too often costly at that, can alone decide the effect of the impact, &c., caused by a live load. Its effect is usually included by adding some per

cent. of the live load to the total load regarded as static.

52. The weight of chords, in table above, is greater than usual perhaps, as it is not generally customary to find the maximum strain in the chords as above. If two opposing trains meet in the center of the span or elsewhere on the span, the strains induced would be greater than given by our formulæ if the center driving wheels of the two locomotives are less than 50 feet distant. The conditions, included in the formula for chords, are that cars may precede and follow engines 50 feet apart, a condition that certainly can be realized in practice. In fact the end panels would be strained nearly as given by our formula, with engines in front as usual.

The maximum chord strains thus found will however be more rarely felt than the max. web strains, for the latter are caused by every passage of the supposed train—engines in front—whilst the former are only felt when the engines are in the *midst of* a train. Let us conceive the

whole live load *uniformly* distributed over the bridge; the total panel weight then would be 35666 lbs., which substitute for P in the formula, art. 39, and make E=*o*. We have,

$$c_n = t_n = \frac{Pl}{2h}(N-n)\ n = 10600\ (N-n)n$$

On computing the various chord strains from this formula and comparing with the *max.* strains previously found, we shall find that we must add 10 per cent. to strains in first two end panels, 9 per cent. for next two panels and 8 and 7 per cent. to the strains in panels next the center, in order that the strains thus found may equal the max. strains.

For the simple trusses just examined, the determination of max. chord strains is simple, but for compound trusses with two or more systems of triangulation, the method is tedious comparatively, and in practice it would be best to ascertain and tabulate for various spans and loads, the percentages to add to the strains resulting from the load regarded as uniformly distributed.

53. *Permissible Strains per Square Inch in Tension and Compression.*—In Van Nostrand's Magazine for Nov. 1877, p. 459, is an article by the writer on this subject. A brief summary of it will be given.

Weyrauch (see "Constructions of Iron and Steel," Chap. XIII) deduces from Wohler's experiments, by Launhardt's formula, the following value for the safe strain in kilograms per square centimeter $=b'$ to which wrought iron should be subjected in tension.

$$b' = \frac{2100}{n}(1 + \tfrac{1}{2}\theta)$$

where $n$=factor of safety, $\theta = \dfrac{\text{min. B}}{\text{max. B}} =$ minimum strain that piece ever bears / maximum strain that piece ever bears

*Impact*, vibration, &c., such as a live load causes is not included; and I assumed that its effect varied inversely with $\theta$, and wrote empirically, for *the safe strain on wrought iron ties in lbs. per square inch*,

$$b = 7500\ (1+\theta) \quad . \quad . \quad (7)$$

Also, the *safe strain on wrought iron columns in lbs. per square inch,*

$$b = \frac{1}{4+\frac{1}{10}\left(\frac{l}{d}\right)} \frac{38500}{1+c\left(\frac{l}{r}\right)^2}(1+\theta) \quad . \quad (8)$$

where $c = \frac{1}{36000}$ for pillars with *flat ends*, $c = \frac{2}{36000}$ for *both ends hinged*, and $c = \frac{1}{24000}$ for *one end flat, the other hinged;* $l$ = length of pillar in inches; $d$ = diameter in direction of bending in inches, and $r$ = radius of gyration of cross section about neutral axis in inches. The factor, $\frac{38500}{1+c\left(\frac{l}{r}\right)^2}$ is supposed to be the crippling weight of the column. This term is found *not* to be constant for different forms of cross section as "square column," "Phoenix," "American" or "common" column.* It would be proper then to replace 38500 and the values given above or found experimentally for

* See *Engineering News* (Chicago), January 31, 1878, for proposed constants in Gordon's formula.

$c$, by the corresponding terms for the particular cross section as found from experiment.

54. The above formulæ cause $b$ to diminish for web members more rapidly towards the center of the span than Weyrauch's formulæ do. As impact is more hurtful the smaller the member and as the weight of web members diminishes towards the center of the span this appears reasonable. Should we assume that $b$ from impact alone varied with the weight of the web member of a bridge, the result would be somewhat different from the above, since the weight of web members increases pretty regularly from the center to the abutments, whereas $\theta$ increases most rapidly at first.

55. For the chords it will be sufficiently near to put

$$\theta = \frac{\text{dead load of bridge}}{\text{total dead and live load}}$$

Thus in the example art. 42 for chords

$$\theta = \frac{336000}{336000 + 520000} = \frac{336}{856} = .39$$

If the chord strains are determined by supposing the bridge *uniformly loaded*, then $\theta$ is correctly determined as above.

56. Since the strain on any web member is equal to the shearing force on that member multiplied by sec. $i$,

$$\theta = \frac{(\text{min. S}) \sec. i}{(\text{max. S}) \sec. i} = \frac{\text{min. S}}{\text{max. S}'}$$

for web members.

Then, arts. 26 and 27, and table art. 21 we get

| Panel | Shearing Forces | | $\theta$ |
|---|---|---|---|
| | Max. | Min. | |
| 1 | 216108 | 77000 | .36 |
| 2 | 181840 | 59112 | .32 |
| 3 | 148960 | 39836 | .27 |
| 4 | 117468 | 19172 | .16 |
| 5 | 87364 | — 5380 | 0 |
| 6 | 58648 | —31320 | 0 |

As given in the table art. 42.

If the strains on the web members are known, they may be used in place of the corresponding shearing forces, if preferred.

57. We see that for a 200′ span bridge weighing 336000 lbs., that $b$, for tension, is varied from 7500 lbs. per square inch on counters and middle ties to 10420 lbs. per square inch for lower chords. Extending the formulæ now to other spans, we should similarly find that for spans of 0, 100, 200, 300, 400 feet, $b$ would vary from 7500 lbs. at center on web ties to 7500, 9400, 10400, 11300, 12200 respectively on end ties or lower chords. When $\frac{l}{d} = 10$, nearly the same figures apply to posts. Thus eqs. (7) and (8) give nearly the same value for all values of $\theta$ when $\frac{l}{d} = 10$.

58. When the engine comes directly on a member, the effect of impact is much greater than for the web members and must be allowed for empirically; thus we have added 50 per cent. to stringers and floor beam loops, the latter because of their small size.

For the floor beams we have supposed $M$ in $B = o$ $\therefore$ $\theta = o$ and $b = 7500$.

For wind strains values of $b$ of 1500 for ties and 5000 for struts was used as the conditions assumed are so rarely fulfilled. It may be remarked that nothing has been added to the chords for wind strains, though its effect must be severe on them, causing *inequality* of strain—another reason why the chords should be computed for maximum strains as in art. 40.

59. The above formulæ (7) and (8) may or may not bear the crucial test of practice. It will probably be admitted however that they possess great advantages in properly comparing different forms of trusses of the same span, to which use they will be put in what follows. Empirical rules in ordinary use are wanting in this; they do not recognize Wohler's law—that the minimum strain sufficient for rupture *decreases* as the difference between the extremes of strain to which the piece is liable, *increases*.

The deduction of Launhardt from Wohler's experiments, that $b$ varies with

$\theta$ is included in the formulæ above; and the coefficient of $\theta$ was changed from $\frac{1}{2}$ as given by Weyrauch to 1 to allow empirically for impact.

60. Gerber also deduced formulæ from Wohler's experiments including 50 per cent. added to live load for impact. Formula (7) above agrees very closely with the values *used* in the Mainz bridge by Gerber, though for $\theta=\frac{2}{3}$ to 1 his formula gives much larger values than eq. 7.

When $\theta=1$, impact is supposed null and $b=15000$ lbs. This seems sufficiently large, though Gerber gives in his Mainz bridge and later formulæ, 22760 lbs. per square inch.

61. The variable factor of safety for posts, $\frac{1}{4+\frac{1}{10}\frac{l}{d}}$ is used to give values for a 200 feet span in accordance with the recommendations and usage of American engineers.

No formulæ for wood is given, as no experiments have been made after Wohler's manner upon it.

62. The compression members in the table art. 42 were supposed hollow cylindrical and of wrought iron. Therefore in eq. (8), $r=\frac{d}{4}\sqrt{2}$. The end upper chord panels were regarded as "flat at one end, hinged at the other;" the other panel lengths as "flat at both ends." The braces were regarded as "hinged at both ends."

The panel length assumed may not be the most economical. It is only by computing the whole weight of the bridge for different panel lengths that the proper panel length can be determined. The most economical height of truss will be considered later.

63. The following table of "crippling weights" may prove a convenience:

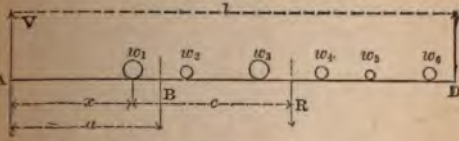
## Hollow Cylindrical Columns.

| $\frac{l}{d}$ | Flat Ends. $\frac{38500}{1+\frac{2}{9000}\frac{l^2}{d^2}}$ | One end hinged. $\frac{38500}{1+\frac{1}{3000}\frac{l^2}{d^2}}$ | Both ends hinged. $\frac{38500}{1+\frac{4}{9000}\frac{l^2}{d^2}}$ | $4+\frac{1}{10}\frac{l}{d}$ |
|---|---|---|---|---|
| 10 | 37663 | 37258 | 36862 | 5. |
| 11 | 37492 | 37008 | 36535 | 5.1 |
| 12 | 37306 | 36736 | 36184 | 5.2 |
| 13 | 37106 | 36447 | 35810 | 5.3 |
| 14 | 36893 | 36139 | 35414 | 5.4 |
| 15 | 36667 | 35814 | 35000 | 5.5 |
| 16 | 36428 | 35473 | 34567 | 5.6 |
| 17 | 36177 | 35117 | 34117 | 5.7 |
| 18 | 35914 | 34747 | 33653 | 5.8 |
| 19 | 35640 | 34365 | 33177 | 5.9 |
| 20 | 35357 | 33971 | 32688 | 6. |
| 21 | 35064 | 33566 | 32190 | 6 1 |
| 22 | 34761 | 33152 | 31684 | 6.2 |
| 23 | 34450 | 32729 | 31171 | 6 3 |
| 24 | 34131 | 32298 | 30653 | 6.4 |
| 25 | 33805 | 31862 | 30130 | 6.5 |
| 26 | 33472 | 31420 | 29605 | 6.6 |
| 27 | 33133 | 30973 | 29079 | 6.7 |
| 28 | 32787 | 30523 | 28551 | 6.8 |
| 29 | 32438 | 30070 | 28025 | 6.9 |
| 30 | 32083 | 29615 | 27500 | 7. |
| 31 | 31725 | 29159 | 26977 | 7.1 |
| 32 | 31363 | 28702 | 26458 | 7.2 |
| 33 | 30998 | 28246 | 25943 | 7.3 |
| 34 | 30631 | 27791 | 25433 | 7.4 |
| 35 | 30262 | 27337 | 24928 | 7.5 |
| 36 | 29891 | 26885 | 24428 | 7.6 |
| 37 | 29520 | 26436 | 23936 | 7.7 |
| 38 | 29147 | 25990 | 23450 | 7.8 |
| 39 | 28774 | 25547 | 22971 | 7.9 |
| 40 | 28402 | 25109 | 22500 | 8. |

64. *Maximum Chord Strains due to any number of equal or unequal weights placed at fixed distances apart.*

Let $w_1, w_2 \ldots$ at fixed distances apart be placed on the girder AD, of span $l$. Let $R = w_1 + w_2 + m = \Sigma w$, be the resultant of $w_1, w_2 \ldots$ in position and magnitude.

FIG. 8*a*.




Call $x$ the distance from A to $w_1$, $c$= distance from $w_1$ to R, $a$=distance from A to the cross section *whose max. moment, as the load moves forward, is required.* We have $Vl = R(l - x - c)$.

1/. *When $w_1$ and $w_2$ are on either side of* B, the moment at B is

$$M = Va - w_1(a - x) = \left\{ R\left(1 - \frac{c}{l}\right) - w_1 \right\} a + \left(w_1 - R\frac{a}{l}\right)x \quad . \quad . \quad . \quad . \quad . \quad (9)$$

When $\left(w_1 - R\frac{a}{l}\right) > o$, M increases with $x$;

*i.e.*, M is a max. for $x=a$. (We must not consider $x>a$, since V and hence M would be diminished). If $\left(w_1-R\frac{a}{l}\right)>o$, M increases as $x$ decreases, which moves $w_2$ up to B at least. When the coefficient of $x$ is zero, $w_1$ or $w_2$ may be supposed at B, or B must lie between them.

2/. *But* M *may be a max. when* $w_2$ *is to the left of* B. In this case regard $w_1+w_2=P$ as a a single force.

Call $c_1$=distance from the center of gravity of $w_1$ and $w_2$ to R, *i.e.*, from P to R,

and $x$=distance from A to P.

Then the above equations hold, on simply substituting P for $w_1$.

As before, when $\left(P-R\frac{a}{l}\right)>o$, M increases with $x$ and is a max. when $w_2$ is over B, but it must not be supposed to the right of it, as the equation does not now include this supposition. This position of the load then gives M a *max.*

3/. If, however, $\left(P-R\frac{a}{l}\right)<o$, M in-

creases as $x$ decreases, whence the load is moved forward so that $w_2$ rests upon B.

Next, calling $w_1+w_2+w_3=P_1$, $c_2=$ distance from $P_1$ the resultant of $w_1,w_2,w_3$ in position and magnitude to R, and $x=$ distance from A to $P_1$; eq. (9) holds as before on substituting $P_1$ for $w_1$. We proceed as before to ascertain if M is a maximum when B is between $w_3$ and $w_4$ and so on for other positions of the loads.

65. As $w_1$, $w_2$ . . . pass off the span, they must no longer be included in the formulae for R, $c$, M, etc. For a framed truss, $a$ is the distance from A to the apex that is taken as the center of moments for the opposite chord panel. As it is only necessary to consider half a truss, the maximum strains in the chords being the same for the other half when the load moves in an opposite direction, *we must not take* $a>\frac{1}{2}\,l$.

66. *Example.* Consider the three trusses, Figs. 5, 6 and 7, to be of 400 feet span, each with 20 panels, the panel lengths *thus being* 20 feet each. Let

five equal weights $w$ (as the locomotive excesses, art. 16, 38) be placed on the span at equal or unequal distances apart. Then $R=5w$ and in eq. (9), $\left(w_1-R\frac{a}{l}\right)$ $=w\left(1-\frac{5a}{l}\right)$, which is positive when $a<$ $\frac{l}{4}=80$ feet; hence art. 64, 1, to find the maximum chord strain on the first four panels from the abutment, the loads $w$, $w$ . . . must extend from the center of moments for the panel considered (art. 65) towards the center. For the fifth chord panel $\left(1-\frac{5a}{l}\right)$ is negative, so that the second weight moves up, at least, as far as its center of moments; then proceeding as in art. 64, 2; $P=2w$, and $\left(P-R\frac{a}{l}\right)=w\left(2-\frac{5a}{l}\right)$ is $+$, so long as $a<\frac{2}{5}l=160$ feet. So that for panels 5, 6, 7, 8 (and 4 if preferred), the second weight must be supposed up to the panel considered, to ascertain its maximum chord strain.

Finally, $\left(2-\frac{5a}{l}\right)<o$ for $a>160$, or 8 panel lengths. Therefore in art. 64, 3, put $P_1=3w$ $\therefore$ $\left(P_1-R\frac{a}{l}\right)=w\left(3-\frac{5a}{l}\right)$ is + when $a<\frac{3l}{5}=240$ feet.

Hence for panels 9, 10, (and 8 if desired) the middle weight is placed at their center of moments.

The above results are independent of the distances apart or magnitude of the equal weights. It is seen that the maximum moment for an end panel is when the front weight reaches to it; whilst the max. moment at the center is when the middle weight is at the center, and for intermediate panels the loads have intermediate positions. It will be instructive for the reader to test the above results, by assuming various positions of the loads for each panel in turn.

When the loads are unequal the application is equally simple and direct.

67. Referring to eq. (9), and regarding $w_1$ as the resultant of all the weights

to the left of B, we see (as was remarked in art. 64) that *when* $a$ has such a value that,

$$w_1 - R\frac{a}{l} = o \therefore w_1 l - (w_1 + w_2 + \dots)\, a = o$$

$$\therefore \frac{w_1}{w_2 + w_3 + \dots} = \frac{a}{l - a},$$

that the greatest moment ever experienced at B obtains; and we see from the last eq. that this occurs when *the loads on either side of B are in the ratio of the segments into which it divides the span.* This conclusion is reached in DuBois, Graphical Statics, art. 73. Though this author seems to regard the analytical treatment of a given recurring system of moving loads as almost impracticable, (see his Preface, p. xi.)

It is believed that the above solution is practical and simple; in fact, much more so than the one by the graphical analysis.

COMPOUND SYSTEM.

68. As the span increases, the panel lengths become too long for economy,

or the inclination of the web diagonals, for usual panel lengths, is not the best for economy, for the trusses previously figured. Hence the use of compound systems such as the whipple, Fig. 9, the Trellis, the Post, or even bridges of "treble" &c. "intersections," where the ties cross three or more panels.

69. *Web Strains.*—In the truss Fig. 9 of 200 feet span and 28 ft. high, divided into 12 panels, weights as before, let *w*, placed below the apices, denote the *panel dead load; p* placed above the apices, the *panel car load;* and E, the *locomotive excess*, being the two weights at *d* and *h* in the figures. It will be noticed that any weight as that at *f* can travel to either abutment only by one web system, as *a* B *d* D *f* H *h* J . . . . The weight at *e* must follow the other system, *a* B *c* C *e* . . . . The weights inked black thus travel towards either abutment only by the first system, the others by the second.

Now if the second engine is placed 50 back of the first, its position is *g;* but

the interposition of a car $16\frac{2}{3}$ feet
or different engine and tender le
would locate it at $h$. As the object
find the max. strain that can come
each system in turn, we must plac
second locomotive four panel le
from the first, so that it will bear
the same system as the first. This
tion may rarely happen, but it shou
provided for in the sections of the
members, especially those near the
ter of the truss, such as the d
"counters." The dotted lines $\overline{Bl}$
$\overline{Ll}$ are "suspenders" like those in I
and similarly strained to a max. of
pounds.

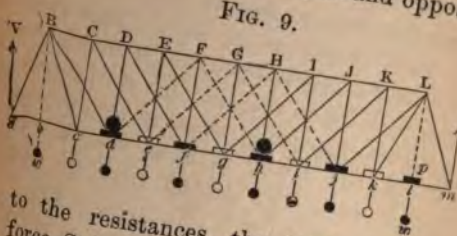
70. The weight at $l$ may be take
either or both of the partial trusse
which we shall suppose Fig. 9 div
As it only affects V, $\frac{1}{12}(w+p)=$
pounds in this instance, it may act
a different system from that taken,
out altering the sections an appre
amount.

With vertical end posts there
uncertainty, for then the weight at

only act with the *black* system, as a tie extends from *l* to top of end post, the tie from *k* being also carried there.

71. If we cut the truss through between $\overline{cC}$ and $\overline{dD}$ as in art. 7, and apply forces at the cut parts equal and opposed

FIG. 9.




to the resistances, the total shearing force, S, is, of course, $V-2w$; but S is the *sum* of the vertical components on the two cut ties, and without knowing the amount carried by one tie we are unable to estimate it on the other. Hence this general method must first be applied to one partial truss and then the other independently. First take the partial truss $aBdDfFh\ldots$, and call V the reaction at *a* due to the load on this truss. The reasoning of arts. 12 and 27 apply here in finding max. and min. S.

Call $\Sigma w$ the *sum* of the $w$'s between $a$ and foremost locomotive on system taken; then having found V, when load, (engines in front) extends from farthest abutment we have

$S = V - \Sigma w$. Take moments about $m$,

Call $l_1$, the lever arm of E,

$l_2$, the lever arm of the resultant of the car loads $p$,

and $m$=the number of $p$'s on the partial truss considered. Also let a panel length as $\overline{ab}$ be the unit of length. With the loads as in the figure $l_1=7$, $l_2=5$, $m=5$. We have as before, $w=14000$, $p=\frac{50000}{3}$ and E=60000.

$$V\,12 = 6\,w\,6 + E l_1 + mpl_2$$

$$\therefore\ S = 42000 + 5000.l_1 + m.1390.l_2 - \Sigma w.$$

In a similar manner we proceed for the other partial truss, finding the equation,

$$S = \tfrac{5}{2}\,w + \frac{E}{12}\,l_1 + \frac{mp}{12}\,l_2 - \Sigma w.$$

72. The results are entered in the following table. Note, from the figure, that as the live load moves two panels to *the right*, $l_1$ diminishes by 2, $l_2$ by 1, and

## Partial Truss $aBdDfF$ . . . .

| Front Engine at | Piece. | $42000+5000.l_1+m.1390.l_2-\Sigma w=$ | S. |
|---|---|---|---|
| $d$ | B$d$ | $42000+5000.7+5.1390.5-0$ | 97750 |
| $f$ | $d$D, D$f$, | $42000+5000.5+4.1390.4-14000$ | 61240 |
| $h$ | $f$F, $h$F, | $42000+5000.3+3.1390.3-28000$ | 27510 |
| $j$ | $h$H, $j$H, | $42000+2500.3+2.1390.2-42000$ | − 940 |
| $l$ | — | $42000+2500.\ +\ 1390\ -56000$ | −24110 |

## Partial Truss $aBcCeE$ . . . .

$35000+5000.l_1+m.1390l_2-\Sigma w$

| | | | |
|---|---|---|---|
| $c$ | $c$B | $35000+5000.8+5.1390.6-0$ | 116700 |
| $e$ | C$c$, C$e$ | $35000+5000.6+4.1390.5-14000$ | 78800 |
| $g$ | $e$E, $g$E | $35000+5000.4+3.1390.4-28000$ | 43680 |
| $i$ | $g$G, $i$G | $35000+5000.2+2.1390.3-42000$ | 11340 |
| $k$ | — | $35000+2500.2+\ 1390.2-56000$ | −13220 |

$m$ by 1. The table is thus very quickly formed. The max. shearing force on $\overline{aB}$=216108 lbs., found by supposing the whole bridge loaded as in art. 21. Thus

$$V=S=\tfrac{11}{12}(w+p)+9\tfrac{1}{2}\times\frac{E}{12}=216108.$$

73. Be careful to note when one locomotive leaves the bridge and modify the formula correspondingly. S sec. $i$ gives the strain on the ties as before, sec. $i=\frac{43.5}{28}=1.553$ except for B$c$; its value for that tie being $\frac{32.5}{28}=1.165$. The values of S above are the actual strains on the vertical posts opposite them. If those posts were inclined as in the "trellis" bridge (in which the posts $\overline{Dd}$, $\overline{Ff}$, &c., take the positions, $\overline{Ed}$, $\overline{Gf}$, &c.; the corresponding ties $f\overline{D}$, $h\overline{F}$, &c., the positions $f\overline{E}$, $h\overline{G}$, &c.; braces reaching from C to $b$ and from $\overline{K}$ to $l$), we multiply S by sec. $i$ to find the strains on them as well as the ties that are equally inclined. In the trellis, as in the triangular, some web members near the center take ob*verse and* reverse strains.

74. It is implied in the previous computations that the reactions at the abutments of each partial truss are to be determined by the simple law of the lever *independently* of the other partial truss.

*Unless the counter rods have little or no strain on them for a uniform load this assumption may be incorrect.*

To prove it: take the extreme case, that the main ties $\overline{Bc}$, $\overline{Ce}$, $\overline{Eg}$, are too long to be in action, and that counters, from $c$ to E, $\overline{eG}$, $\overline{gI}$ are *taut* (malicious or ignorant persons might screw up the counters so as to relieve the main ties above from strain), then the loads $c$, $e$ and $g$ must inevitably go to $m$. Now conceive panel $c$D severed. The total shear on this panel (art. 71) is,

$$V-2w=\frac{11w}{2}+\frac{7E}{12}+\frac{9p}{12}5-2w=146500=S.$$

But since the shear $w$ on the supposed rod $\overline{c\,E}$ acts up, the actual shear on tie $\overline{B\,d}$ is $S+w=160500$ lbs.; *whereas we shall proportion it for* 97750 *lbs. shear as previously found!* Similarly for other

panels. Practically, the counters are loose when the truss is first set up; the main ties are then necessarily in action; for even if a little long ($\frac{1}{64}$ inch say), the roadway sinks the apex and thus brings them into action (as the upper chord apex sinks too, the chord at this apex is thus unequally strained). Now, if the counters are tightened, *part* of the weight at *c*, *e*, . . . *will go* to *m*, from the law of decomposition of forces; and *the greater the initial strain on the counters, the larger the weight on the partial truss*, *a*BcCeE . . . *that is transferred to the right abutment.* The counters being of much lighter section than the main ties will stretch more and thus counteract this tendency, especially on the partial truss on which the locomotives bear.

95. As stated then, the partial trusses cannot act independently except when the counters are not strained for a uniform load. If they have a slight initial strain on them, the strains may be increased on the web members somewhat—an additional reason for supposing the

second locomotive to bear upon the same system as the first, as we have done. It is then not only useless, but may be prejudicial to put counter rods, screwed taut, in panels where there is never any reversion of strain, as from *c* to E etc. With the loads assumed, S is—, (art. 73) for counters *d* F and *j* H, but a slightly greater load would bring them into action, hence they should be retained and their section assumed, say at 2 sq. in.

76. The preceding reasoning applies to all compound systems. In finding S for the simple systems, as Figs. 5, 6, 7, no assumption was made as to the abutment to which any particular weight was transferred. But S is the total shear, and for Figs. 5 and 6, if the counter carries any strain (as explained in art. 74) the shear on the main tie or brace is increased by that strain.

In the triangular truss, Fig. 7, there is no uncertainty as to the strains; as the same piece, where necessary, takes tension and compression both.

77. It is recommended as good prac-

tice, where separate counter rods are used, "to put a *light* load on the bridge and then strap the counters down taut. They should not remain taut under full symmetrical loads, but should be tight enough to keep quiet under unsymmetrical loads; a medium that can be struck." "Practically the counter ties remain tight if adjusted intelligently and are not tampered with." Similar remarks apply to "keying wooden counterbraces" recommended by Haupt. As any deflection of a truss is accompanied by an increased length of main ties, and a shortening of the counter diagonals, if the counters are strapped down when there is a light load on the bridge, for a full symmetrical load they will be loose; for a dead load only, they will be tight. This is as it should be.

(*Remark.*—The whole of the rearmost loc. excess was supposed to bear at one apex only, thus giving slightly larger strains than the true ones, to *allow* somewhat for improperly adjusted counters.)

78. *Chord Strains.*—Suppose the truss loaded at each lower apex with $(w+p) = w'$ (the weights below the Fig. can now be taken for $w'$) and of the two weights 30,000 lbs. each, either 3 panels (50 ft.) apart, or a greater distance, if the chord strains are thereby increased.

As before, we assume that each partial truss acts independently of the other; find the strains on any chord piece as $\overline{CD}$, due to each partial truss and add them for the total strain on that member. If it is simpler, the reader may draw the two partial trusses separately to estimate the effect of each.

79. *The Locomotive Excess*, E=60,000 lbs., is made up of the two weights, 30,000 lbs. each. For convenience call the foremost P, the rearmost, P′.

Now the chords may receive their maximum strains when P and P′ act in the same system, 4 panels apart; or in different systems, 3 panels apart. The principles of art. 34 are of some assistance, but we can only determine by actual trial, for each chord panel in turn,

the proper relative position of P and P′ that give the max. strains for that panel. Hence I have estimated the effect of P and P′ separately, and have taken those positions of P and P′, *either* 3 *or* 4 *panels apart*, that gave the greatest strains for each chord panel. The height of truss was taken as before at 28 ft. The results are as follow:

| Piece. | P at | P′ at | Max. Strains. | P, 4 panels from P′. |
|---|---|---|---|---|
| | | | lbs. | lbs. |
| *ac* | *b* | *e* | 28274 | — |
| *cd* | *c* | *g* | 47619 | 47619 |
| *de* or BC | *d* | *h* | 62500 | 62500 |
| *ef* or CD | *d* | *g* | 75893 | 71433 |
| *fg* or DE | *e* | *h* | 84821 | 74410 |
| EF | *f* | *i* | 87798 | 71433 |
| FG | *f* | *i* | 87798 | 71433 |

Thus for $\overline{cd}$, $\overline{de}$ and $\overline{BC}$, the weights are 4 panels apart, for the others 3. Note again, that for some panels P is adjacent, and for others a panel distance from, the panel considered. It will be noticed above that for panels near the centre the strains are considerably great-

er when P and P′ are 3 panels apart, than when they are 4 panels apart.

80. To illustrate the method of computation, call *a panel length*$=ab=1$; the height of the truss is then $28 \div \frac{5}{3}0 = \frac{8}{5}4$. With P at $c$, its reaction at $a$ is $\frac{1}{1}\frac{0}{2}$ P. If $\overline{de}$ is cut, rotation about C would occur. The moment about C is $\frac{1}{1}\frac{0}{2}$ P. $2 = 50000$. Next suppose P at $d$, the reaction is $\frac{3}{4}$ P. If $\overline{de}$ is cut, D is the point of rotation, and the moment is thus $\frac{3}{4}$ P $3 = 67500$.

Now when P is at $c$, conceive P′ at $f$; its reaction is $\frac{7}{1}\frac{}{2}$ P; and with D as a center of moments for $\overline{de}$ cut the moment $= \frac{7}{1}\frac{}{2}$ P $3 = 52500$. (It is needless to consider P′ at $g$, in this case, as V is less, also the point of rotation, for $\overline{de}$ cut, being C, for the truss $g$ E $e$ $l'$ . . ., the lever arm is less too).

Next with P at $d$, let P′ be at $g$ ∴ V$=\frac{1}{2}$ P. Then for $\overline{de}$ cut, the moment about C is $\frac{1}{2}$ P. $2 = 30000$.

Lastly with P at $d$, conceive P′ at $h$ ∴ V$=\frac{5}{1}\frac{}{2}$ P. For $\overline{de}$ cut, D is the point of

rotation for truss $h$ F $f$ D . . . ., hence the moment is $\frac{5}{12}$ P. 3=37500.

Collect now the moments for the piece $\overline{de}$.

| Piece. | P at | Moment | P′ at | Moment | Total. |
|---|---|---|---|---|---|
| *de* | *c* | 50000 | *f* | 52500 | 102500 |
| *de* | *d* | 67500 | *g* | 30000 | 97500 |
| *de* | *d* | 67500 | *h* | 37500 | 105000 |

With P at $d$ and P′ at $h$, the actual moment is greatest. Divide it (105000) by the height of truss $\frac{84}{50}$ and we get the strain in $\overline{de}$=62500 lbs.

81. We see how much simpler the treatment of the simple systems is than the compound; still, if we desire to know the "true inwardness" of the compound systems, extra work is unavoidable.

It may be urged that when P′ passes to the right of all the counters none of its weight can be transferred to *a*: true, but with the uniform load in addition on the bridge, the law of the lever holds for each partial truss, since counters are *designed* in those panels where loads

have to be transferred to the farthest abutment and the *effect must be the same in the final summation whether the two engines and the uniform load are treated separately or conjointly.*

82. *For the Uniform Load,* $w' = w + p = 14000 + 16666 = 30666$ per panel. As before we *assume* that the partial trusses act independently and afterwards combine their effects for the same chord panel.

Thus to find the strain on $\overline{BC}$ due to the black weights: $V = \frac{1}{2}6\, w'$, with $d$ as a center of moments,

$$(\text{Strain on } \overline{BC}) \times \overline{dD} = V \times \overline{ad} - w' \times \overline{bd}$$

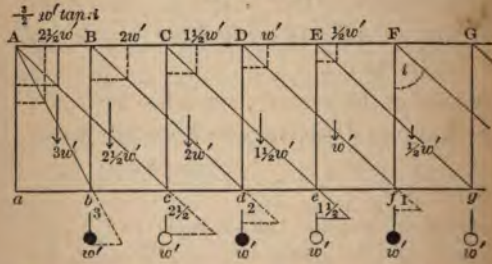
Similarly for the other partial truss, conceive $\overline{bc}$, $\overline{Bc}$ and $\overline{BC}$ cut and take the intersection of the first two, $c$, as a center of moments (art. 36). The reaction at $a$ is $V' = \frac{5}{2} w'$

$$\therefore (\text{Str. on } \overline{BC}) \times \overline{Cc} = V' \times \overline{ac}$$

The sum of the strains on $\overline{BC}$ thus found, added to that found for the locomotive excess, gives the total strain on $\overline{BC}$ which is evidently the same as that *on* $\overline{de}$.

83. The following is a more convenient method. The reaction at $a$ due to the black weights is $3w'$; hence (art. 7), the *shears* on $\overline{b}\mathrm{A}$, $\overline{d}\mathrm{B}$, $\overline{f}\mathrm{D}$ are $3w'$, $2w'$, $w'$, respectively as is marked, on the half truss with vertical end-posts, Fig. 10.

FIG. 10.




The shearing forces on the ties of the other partial truss are as marked on them $2\frac{1}{2}w'$, $1\frac{1}{2}w'$, $\frac{1}{2}w'$. In fact $\frac{1}{2}$ of the $w'$ at $g$ (Fig. 9) goes to either abutment (if the counters do not act). At E the pull on the tie $g\,\mathrm{E}$ is decomposed into $\frac{1}{2}w'$ acting down the post $\mathrm{E}e$ and $\frac{1}{2}w'$ tan. $i$, compressing EF, ($i = g\,\mathrm{E}e$).

At $e$, the $\frac{1}{2}w'+w'$ (at $e$) acting vertically, is decomposed in the directions

$\overline{Ce}$ and $\overline{ef}$, thus giving the strain on $ef = 1\frac{1}{2}w$ tan. $i$. The shearing force on $\overline{Ce}$ is thus $1\frac{1}{2}w'$. Hence the pull on $\overline{Ce}$ at C gives $1\frac{1}{2}w'$ tan. $i$, strain on CD, and $1\frac{1}{2}w'$ strain on post $\overline{Cc}$; and so on for all the weights, except that the pull on A$b$, causes a strain on AB of $3\,w'$ tan. $a\mathrm{A}b = \frac{3}{2}w'$ tan. $i$. Put $a\mathrm{A}b = i_1$. The total strain on AB $= (\frac{3}{2} + 2\frac{1}{2})w'$ tan. $i = 3w'$ tan. $i_1 + 2\frac{1}{2}w'$ tan. $i$.

Strain on BC = strain on AB $+ 2w'$ tan. $i$.
Strain on CD = strain on BC $+ 1\frac{1}{2}w'$ tan. $i$.
&c., &c.

Similarly,
strain in $\overline{bc} = 3w'$ tan. $i_1 = \frac{3}{2}w'$ tan. $i$.
strain in $\overline{cd}$ = strain in $\overline{bc} + 2\frac{1}{2}w'$ tan. $i$, &c.
Hence the rule.

*Multiply the shear on each inclined web piece by the tangent of its inclination to the vertical. The summation of these products from the abutment to any chord piece gives its total strains.*

If the chord strain at center agrees with that found by moments the whole work is correct. This method can be applied to *any truss.*

83. Now by the principle of moments, the expressions for the strains in $\overline{BC}$, $\overline{CD}$ . . . , $cd$ . . . are the same for Figs. 9 and 10. In fact the same method may be applied to Fig. 9, regarding the inclination, &c., at the ends, and the above rule deduced. The strains for the truss Fig. 9 are entered in the following table. By computation, we find, tan. $i$=1.19; tan. $i_1$=.595.

84. The total strain on $\overline{ac}$ Fig. 9 is the same as for Fig. 7, 128669 lbs. Since the shear on $a$B is $\frac{1}{2}w'$ we find the strain on $ac$ due to uniform load $\frac{1}{2}w'$ tan. $i_1$=100354.

From table, art. 79, the max. strain due to E is 28274 which gives 128628 lbs. strain on $\overline{ac}$. The difference, 41 lbs., between this result and the former is due to carrying *tan. i* to two decimal places only.

| Piece. | | Increments. | Strains. |
|---|---|---|---|
| *bc* | $5\frac{1}{2}\,w'$ tan. $i_1$ | 100354 | 100354 |
| *cd* | $2\frac{1}{2}\,w'$ tan. $i_1$ | 45616 | 145970 |
| *de* or BC | $2\,w'$ tan. $i$ | 72984 | 218954 |
| *ef* or CD | $1\frac{1}{2}\,w'$ tan. $i$ | 54738 | 273692 |
| *fg* or DE | $w'$ tan. $i$ | 36492 | 310184 |
| EF or FG | $\frac{1}{2}\,w'$ tan. $i$ | 18246 | 328430 |

Taking moments about $g$ we have

$$\text{Strain in } \overline{FG} = (1\tfrac{1}{2}\,w.100 - 5w.50) \div 28$$
$$= 328550 \text{ lbs.}$$

The slight difference between this result and that given in the table, shows the correctness of the work. The "increment" column can be "run up" from the bottom, adding $\frac{w'}{2}$ tan. $i$ each time until we reach $\overline{cd}$.

85. Combining these results with those in art. 79 we enter them, also the web strains in the following table:

| Piece. | | $d$. | $\frac{l}{d}$ | th | Strain. | $\theta$. | $b$. | Area. | Length. | No. | $k$. | Weight. | Totals. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | ″ | | ″ | | | | □″ | ′ | | | lbs. | |
| U. Chord, | BC | $13\frac{1}{8}$ | 15 | $\frac{3}{4}$ | 281454 | .39 | 9050 | 31.1 | $\frac{100}{6}$ | 4 | $\frac{10}{3}$ | 6911 | |
| | CD | “ | “ | 1 | 349585 | “ | 9270 | 37.7 | “ | “ | “ | 8378 | |
| | DE | “ | “ | $1\frac{1}{8}$ | 395005 | “ | “ | 42.6 | “ | “ | “ | 9467 | |
| | EF | “ | “ | $1\frac{1}{4}$ | 416228 | “ | “ | 44.9 | “ | “ | “ | 9978 | |
| | FG | “ | “ | $1\frac{1}{4}$ | 416228 | “ | “ | 44.9 | “ | “ | “ | 9978 | 44712 |
| Posts, | $aB$ | $13\frac{1}{16}$ | 30 | $\frac{21}{16}$ | 251766 | .36 | 5340 | 47.15 | 32.6 | “ | “ | 20495 | |
| | $Cc$ | 10 | 34 | $\frac{20}{16}$ | 78800 | .17 | 4400 | 17.9 | 28 | “ | “ | 6683 | |
| | $Dd$ | “ | “ | $\frac{19}{16}$ | 61240 | 0 | 3750 | 16.3 | “ | “ | “ | 6085 | |
| | $Ee$ | “ | “ | $\frac{18}{16}$ | 43680 | 0 | “ | 11.62 | “ | “ | “ | 4338 | |
| | $Ff$ | $8\frac{3}{32}$ | 40 | $\frac{17}{16}$ | 27510 | 0 | 3140 | 8.76 | “ | “ | “ | 3270 | |
| | $Gg$ | “ | “ | “ | 11340 | 0 | — | 8.76 | “ | 2 | “ | 1635 | 42506 |
| Lower Chord | | | | | 1476889 | .39 | 10420 | 141.73 | $\frac{100}{8}$ | 4 | “ | 31496 | 31496 |
| Suspender Ties, | $Bb$ | | | | 46000 | 0 | 7500 | 6.13 | 28 | “ | “ | 2288 | |
| | $Bc$ | | | | 135955 | .3 | 9750 | 13.94 | 32.6 | “ | “ | 6064 | |
| | $Bd$ | | | | 151806 | .25 | 9370 | 16.2 | 43.5 | “ | “ | 9396 | |
| | $Ce$ | | | | 122376 | .17 | 8780 | 14 | “ | “ | “ | 8120 | |
| | $Df$ | | | | 95106 | 0 | 7500 | 12.7 | “ | “ | “ | 7366 | |
| | $Eg$ | | | | 67835 | 0 | 7500 | 9.04 | “ | “ | “ | 5243 | |
| | $Fh$ | | | | 42723 | 0 | 7500 | 5.7 | “ | “ | “ | 3306 | |

86. The value of $\theta$ for the chords is the same as in the previous truss examined, also for $\overline{a\mathrm{B}}$ and $\overline{b\mathrm{B}}$. From the table of shearing forces (art. 72), we find the following values for $\theta$, according to the principles of arts. 26, 27:

| Piece. | Maximum S. | Minimum S. | $\theta$. |
|---|---|---|---|
| $a$B | 216108 | 77000 | .36 |
| $c$B | 116700 | 35000 | .3 |
| $d$B | 97750 | 24110 | .25 |
| C$e$, C$e$ | 78800 | 13220 | .17 |
| D$d$, D$f$ | 61240 | 940 | 0 |
| E$e$, E$g$ | 43680 | 11340 | 0 |

The black weights were regarded as acting on the same partial truss. Min. S on $\overline{c\mathrm{B}}$ is then due to dead load only, and is 35000 lbs.$=2\frac{1}{2}w$. Min. S on $\overline{d\mathrm{B}}$ is the same as for $\overline{\mathrm{L}j}$ when front engine is at $l$ (art. 72). Similarly min. S on $\overline{\mathrm{C}e}$, $\overline{\mathrm{C}e}$ is the same as for $\overline{k\mathrm{K}}$, $\overline{\mathrm{K}i}$ when "front engine is at $k$." The dotted counters and hence the posts $\overline{\mathrm{F}f}$ $\overline{\mathrm{G}g}$ sustain no strain from a uniform load. Hence for them $\theta=o$. It will be noticed that for the *same* panel $\theta$, and hence b

is less for the "compound" than for the "simple" systems. In the foregoing table the *posts* were regarded, as "hinged at one end."

87. The following is the

BILL OF MATERIALS.

Whipple Truss—200′ span—28′ high.

| | lbs. |
|---|---|
| Posts | 42506 |
| Upper chord | 44712 |
| 20 p. c. for castings, &c | 17444 |
| Ties, counters and suspenders | 44306 |
| Lower chord | 31496 |
| 15 p. c. on two last, for bolts, &c. | 11370 |
| Floor beam loops | 5000 |
| Lateral bracing | 11400 |
| Floor beams (iron) | 24500 |
| Iron stringers | 60000 |
| Rails, cross ties, &c | 33200 |
| Total weight of bridge | 325934 |
| Assumed weight | 336000 |
| Assumed weight too great by | 10066 |

The weight of this whipple truss (325934) is thus 3915 lbs. less than the weight of the Triangular Truss (329849) allowing $\frac{1}{8}''$ as the least thickness of

metal. If, however, as seems more proper, the vertical posts of the triangular truss that only sustain 2500 lbs. *dead* load, be given a thickness of $\frac{1}{8}$ inch, their section will be 4.5 square inch; and the weight of the triangular truss is reduced 4200 lbs. making it the lightest of the two. The weight of flooring, rails, loops, lateral bracing, etc., was assumed the same in both trusses. See art. 108 for a further comparison.

88. The Quadrangular Truss, however, is more built than any other in this country, on account of its economy in first cost, the *square joints* being more easily and accurately machined than others; the posts too are vertical, thus ensuring less flexure under their own weight than inclined posts, and with certain details they can be made "flat at both ends," bearing against the upper chord and the upper flange of the floor beam.

It is evident from what precedes that "compound systems" require greater accuracy *in fitting* than simple systems,

and where counter rods are used, they should be properly tightened and often inspected, or grave consequences may ensue. It is evident, likewise, that the greater the number of systems used, the more care is required to make the actual strains agree with the computed; in other words, to cause each partial system to act independently of every other. The investigation of the maximum chord strains is more troublesome the greater the number of partial systems used. Many of the largest spans built or being built in this country, varying from 300 to 525 feet in length, are "double intersection," although treble and quadruple intersections are by no means unknown.

In latticed bridges where the diagonals are connected at their intersections. the strains are perfectly indeterminate, It would certainly then seem advisable to use those patterns of web in which the strains go where they are computed to go.

The weights computed above are, so far as I know, above average. Are they

too great for a first-class road? The effects of high speed, with snow, great cold and side wind (for which no provision is made in the chords), ill fittings and perhaps some counters unadjusted should be considered conjointly with the statical loads in answering this question.

89. Let us now suppose the live load uniformly distributed, and ascertain what percentages are necessary to add to the chord strains induced to equal the *maximum* chord strains (see art. 52).

The uniform live and dead load per panel is now $(168000+200000+60000) \div 12=35666$ lbs. which causes the following strains in the chords (see art. 84 for method of ascertaining strains):

BC=254654, EF=381980
CD=318317, FG=381980
DE=360759,

whence comparing with the maximum strains given in the table, we find that for BC and CD, we must add 10 p. c., for DE, 9½ and for EF and FG, 9 p. c. to strains just found to get the corre-

sponding maximum strains. The percentages are greater, except for end panels, than for the simple systems (see art. 52); hence a comparison of weights based on the same percentage, is favorable to the compound system, as drawn in Fig. 9, at least.

90. It is worthy of note that *the strains in the chords are greatest where the shearing force is zero.*

This is evident from the reasoning in art. 83: for as the increment of strain is, *the shear on the tie* $\times$ *tan i; where* the shear on the tie is zero, the chord strain is a maximum. Thus, in Fig. 10, since $s=o$ on tie $\overline{Fh}$, there is no increment of strain to add to the strain on EF, at F. At E, $\frac{1}{2}$ *w tan i* is added to the strain on $\overline{DE}$ etc. We see then, that $\overline{EG}$ is more strained than any other part of the upper chord.

Similarly for irregular loading.

The above result is true, irrespective of the number of panels, hence for an indefinitely great number, as we may suppose a *solid beam* made up of.

This result must not be confounded with that of art. 64, where the object was to find that position of the load for which a particular chord piece would be strained most.

91. Let us now estimate the whipple as a *deck* bridge with leaning end ties, trusses 14′ apart from centre to centre. Thus in Fig. 9 extend the upper chord to the abutments at A and M, discard Bc $a$B and $ab$; draw the ties $b$A, $c$A and the post B$b$ (similarly at the other abutment) and conceive the load on the upper chord. The chord strains are the same as for the through bridge, Fig. 10; the maximum shear on the ties is the same as before, but the maximum strain on a post now is when the front engine is directly over that post, or two panels nearer the abutment than before, thus increasing the strains on the post over those formerly obtained. On this account $\theta$ is not the same for the posts as before. The results are entered in the following abridged table of weights, from which the Bill of Materials is made

| Piece. | $d$ | $\frac{l}{d}$ | th | Strain. | $\theta$. | $b$. | Area. | Length. | No. | $k$. | Weight. | Totals. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | " | | " | | | | | | | | lbs. | |
| U. Chord, $\overline{AB}$ | $13\frac{1}{8}$ | 15 | | 193589 | .39 | 9050 | 20.1 | $\frac{100}{8}$ | 4 | $\frac{10}{3}$ | 4467 | |
| BG | " | " | | 1858500 | " | 9270 | 200.5 | " | " | " | 44556 | 49023 |
| Posts, $Bb$ | 12 | 28 | 1 | 137000 | .3 | 5840 | 23 5 | 28 | " | " | 8773 | |
| $Cc$ | " | " | $\frac{9}{16}$ | 116700 | .3 | 5840 | 20. | " | " | " | 7466 | |
| $Dd$ | " | " | $\frac{1}{2}$ | 97750 | .25 | 5610 | 17.4 | " | " | " | 6496 | |
| $Ee$ | 10 | 34 | $\frac{5}{8}$ | 78800 | .17 | 4400 | 17.9 | " | " | " | 6683 | |
| $Ff$ | " | " | $\frac{1}{2}$ | 61240 | 0 | 3750 | 16.3 | " | " | " | 6085 | |
| $Gg$ | " | " | $\frac{3}{8}$ | 46000 | 0 | 3750 | 12.3 | " | 2 | " | 2296 | 37799 |
| Lower Chord | | | | 1346987 | .39 | 10420 | 129.2 | $\frac{100}{8}$ | 4 | " | 28711 | 28711 |
| Ties, $Ab$ | | | | 159605 | .3 | 9750 | 16.4 | 32.6 | " | " | 7128 | |
| $Ac$ | | | | 181235 | .3 | 9750 | 18.6 | 43.5 | " | " | 10788 | |
| Other ties (as before) | | | | | | | | 43.5 | " | " | 35954 | 53870 |

out as before. To avoid mistake in determining "*min. B*," the partial trusses may be drawn separately, when the principles of art. 27 apply directly:

BILL OF MATERIALS.

Whipple Truss (Deck)—200′ span—28′ deep.

| | lbs. |
|---|---|
| Upper chord and posts..... | 86822 |
| 20 p. c. for castings, &c..... | 17364 |
| Ties and lower chord..... | 82581 |
| 15 p. c. for bolts, &c..... | 12387 |
| Lateral tie rods and struts..... | 11400 |
| 13 floor beams, 24″ deep..... | 22630 |
| Iron stringers, 26″ deep..... | 60000 |
| Rails, cross ties, &c..... | 33200 |
| Total weight..... | 326384 |
| Assumed weight..... | 336000 |
| | 9616 |

MINIMUM MATERIAL.

92. The most economical inclination of the web ties, irrespective of the rest of the bridge, is easily found to be 45°. Thus call $x$ the height of truss, $d$ the horizontal distance between the extremities of the tie, $i$ its inclination to the *vertical, and s* the shearing force on it.

The strain on the tie is thus $s$ sec. $i$; the cross section $s$ sec. $i \div b$, and as its length is $\sqrt{d^2+x^2}$ its volume, is, since sec. $i = \frac{\sqrt{d^2+x^2}}{x}$,

$$V = \frac{S}{b}\frac{d^2+x^2}{x}$$

which is a minimum for $x=d$, or when $i=45°$: *i. e., the material in the web ties is a minimum when they are inclined* 45° *to the vertical.*

93. The same would be true for the web struts, if $b$ was assumed constant for them, but $b$ = the strain per square inch allowed, diminishes with the length of the strut, and the above simple relation does not hold. The general law of maxima and minima is this: that any function of a single variable is a maximum or a minimum for those values of the variable derived by placing the first differential coefficient of the function equal to zero. Thus, $v=$ a maximum or a minimum in the eq. above when

$$\frac{dv}{dx} = \frac{2x^2-d^2-x^2}{x^2} = o \quad \therefore x=d$$

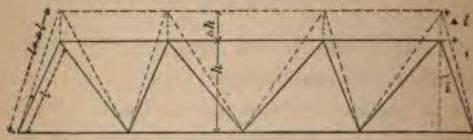
It is evident that $x=d$ gives a min. It is proved by noting that $\frac{d^2v}{dx^2}=\frac{2d^2}{x^3}$ is positive; for as the second differential is $[\pm]$ the function is $\left\{\begin{matrix}\text{a min.}\\ \text{a max.}\end{matrix}\right\}$ for that value of the variable.

94. Having given for a truss with parallel chords, the *span*, *loads*, *panel lengths*, *pattern*, *details*, and *formulæ for* "$b$" (the unit strains), required the most economical height of truss? Denote the weight of the material that varies with the height of the truss (since it is a function of $h$), by F $(h)$. This material is such as given in preceding tables, as computed from the strains on web and chords. The castings, bolts, etc., transverse bracing, flooring system, pins and loops, do not vary perceptibly with $h$, hence F $(h)=$ weight of material computed *on* chord and web strains only; which is easily selected from the table of weights. Now, if F $(h)$ is a minimum we must have

$$\frac{d\,\mathrm{F}h}{dh}=\lim\frac{\mathrm{F}(h+\triangle h)-\mathrm{F}h}{\triangle h}=0 \quad . \quad . \quad (10)$$

95. Denote the weight of upper and lower chords, that varies with $h$, by

FIG. 11.



$W_c$; also denote the variable part of the weight of a web member by $w$, its inclination to the vertical by $i$ and its length by $l$.

Now change the height of the truss (see Fig. 11) to $h + \triangle h$ and call the new value of $l$, $l + \triangle l$.

96. As by assumption, the panel lengths remain the same as well as the diameter of the upper chord, the strains in, and hence the weight of, the chords are now $\frac{h}{h + \triangle h}$ of the first, $h$ and $h + \triangle h$ being respectively the former and present lever arms of the chord strains.

$\therefore$ *New weight of variable material in chords,*

$$W'_c = W_c \frac{h}{h + \triangle h} = W_c - W_c \frac{\triangle h}{h + \triangle h} \quad \ldots\ldots \quad (11)$$

97. If S denote the shear on the web member, whose weight (the part that varies with $h$) was $w$, inclination to vertical $i$ and length $l$, the former strain on it was S sec. $i = S\frac{l}{h}$ and hence its volume was $\frac{S\frac{l}{h}}{b} l = \frac{Sl^2}{bh}$.

The new volume is similarly,

$$\frac{S(l + \triangle l)^2}{b'(h + \triangle h)}$$

in which, for struts (see art. 53)

$$b = \frac{38500(1+\theta)}{\left(4 + \frac{1}{10}\frac{l}{d}\right)\left(1 + c\frac{l^2}{r^2}\right)}$$

and $b'$ equals the same expression on changing $l$ to $(l + \triangle l)$. For ties $b = b' = 7500(1+\theta)$.

Hence *the new weight of the variable material in the web member* = w × ratio *of new to old* volumes

$$= w\frac{(l+\triangle l)^2}{l^2}\cdot\frac{h}{h+\triangle h}\cdot\frac{b}{b'} \quad . \quad (12)$$

Actually dividing $b$ by $b'$; for struts, we get

$$\frac{b}{b'} = 1 + \frac{\frac{\triangle l}{10d}\left(1+\frac{cl^2}{r^2}\right)+\left(4+\frac{1}{10}\frac{l+\triangle l}{d}\right)(2l+\triangle l)\frac{c\triangle l}{r^2}}{\left(4+\frac{1}{10}\frac{l}{d}\right)\left(1+c\frac{l^2}{r^2}\right)}$$

$$= 1 + k\triangle l.$$

For ties this ratio is 1. Hence to avoid complication, simply notice that for ties the fractional term $k$, in the value of $\frac{b}{b'}$ for struts is zero, and as a consequence, when a tie is considered, the term $m$ given below is zero. Now substitute the above value of $\frac{b}{b'}$ in (12) and reduce.

The first term of the 2nd member must now be written,

$$w\frac{(l+\triangle l)^2}{l^2}\cdot\frac{h}{h+\triangle h}$$

$$= w + w\frac{2hl\triangle l + h\triangle l^2 - l^2\triangle h}{l^2 h + l^2\triangle h}$$

So that the new weight of the web member is

$$\left(w+w\frac{2hl\triangle l+h\triangle l^2-l^2\triangle h}{l^2h+l^2\triangle h}\right.$$
$$\left.+w\frac{(l+\triangle l)^2}{l^2}\frac{hk\triangle l}{h+\triangle h}\right)=w'$$

and the sum of these, for the whole web we indicate by $\Sigma w'=\Sigma w+\Sigma$, &c.

98. Now, $F(h)=W_c+\Sigma w$

$$F(h+\triangle h)=W'_c+\Sigma w'$$

$$\therefore \frac{F(h+\triangle h)-F(h)}{\triangle h}=-\frac{W_c}{h+\triangle h}$$
$$+\frac{\Sigma w'-\Sigma w}{\triangle h}$$

Now by the principles of limits; *lim.* $\frac{\triangle l}{\triangle h}=\cos.\ i$, as $\triangle h$ and therefore $\triangle l$ diminish indefinitely.

Hence for a minimum weight of bridge,

$$\frac{dFh}{dh}=\frac{-W_c}{h}+\Sigma\left(\frac{2h\cos.\ i-l}{lh}\right.$$
$$\left.+\cos.\ i\times lim.\ k\right)=o$$

The *lim.* $k$ we find by making $\triangle l=o$

in the value of $k$. Now since cos. $\frac{h}{l}$, the above becomes on multiplyi through by $h$

$$W_c = \Sigma\left(\frac{2h^2}{l^2} - 1 + \frac{h^2}{l^2}(lim.\ k)\ l\right)w$$

Now, $\frac{2h^2}{l^2} - 1 = 2\cos.^2 i - 1 = \cos. 2i$

And

$$(lim.\ k)\ l = \frac{\frac{l}{10d}\left(1 + c\frac{l^2}{r^2}\right) + \left(4 + \frac{1}{10}\frac{l}{d}\right)\frac{2}{}}{\left(4 + \frac{1}{10}\frac{l}{d}\right)\left(1 + c\frac{l^2}{r^2}\right)}$$

Or putting $(lim.k)\ l = m$, and reducing

$$m = \left\{ \frac{\frac{l}{d}}{40 + \frac{l}{d}} + 2\ \frac{c\frac{l^2}{r^2}}{1 + c\frac{l^2}{r^2}} \right\} \ldots (1$$

whence we find the following simple lation:

$$W_c = \Sigma w \left(\cos. 2i + \frac{h^2}{l^2} m\right) \ldots . (1$$

∴ *When the truss has the most economi height, the variable weight of the t chords must equal the sum of all the ter*

*found by multiplying the variable weight of each web member by the cosine of twice its inclination to the vertical plus, a term varying with the ratios of h to l and of l to d; noting that for ties, or posts where b is taken constant, this last term* $\left(\frac{h^2}{l^2} m\right)$ *becomes zero.*

For vertical members, $i=o$, cos. $2i=1$ and $h=l$.

99. For $i<45°$, cos. $2i$ is $+$; $i=45°$, cos. $2i=o$; $i>45°$, cos. $2i$ is $-$.

The above result is true for any pattern of truss whatever with parallel chords *in which* the strain on the chords varies inversely as the heights and the shear on any web member is not altered by a change of height.

The result then, it seems, applies to all usual forms of trusses with parallel chords.

100. Let us draw a few general conclusions from our formula :

1/. The depth of deck bridges with vertical end posts should be less than that of through bridges of *same design*,

since the posts are heavier and thus $W_c$ must be greater to satisfy eq. 14, which requires a lower truss. With no end posts for the deck bridge, if the web is thereby lighter the reverse may be the case.

2/. The greater the number of panels, the heavier the web, for the same height, which requires a lower truss to bring about the equality of eq. 14 (supposing $W_c$ for the same height to be the same for any number of panels, which depends upon the relative unit strains of upper and lower chords).

3/. Continuous girders should have a less depth than simple girders, since for same weight of web, it is known that $W_c$ is less than for a simple girder.

4/. Trusses with two or more web systems should be built deeper than similar designs with one web system, since *cos.* 2 $i$ is nearer $o$ in the first case, hence $W_c$ should be less and the truss higher.

101. If we suppose $b$ (=strain per square inch) *constant* for braces as well as ties $m=o$ and eq. 14 becomes

$$W_c = \Sigma w \cos. 2i \quad \ldots\ldots (15)$$

In Van Nostrand's Magazine for Jan., 1877, p. 42, is an article by Emil Adler, C.E., on the most economical depth of girders, in which he deduces the equivalent of eq. 15. The general method followed above is founded upon that of Mr. Adler; but it will be found that the supposition that $b$ is constant will not give correct results in practice, hence I gave $b$ the variable value, art. 97, and find the results to agree closely with practice. This should be so, since the value of $b$ assumed agrees closely with values now used in America.

102. If all the web members are inclined 45°, as in the Warren girder $\cos. 2i = o$ $\therefore$ $W_c = o$ or the height of the truss is $\infty$. Hence 45° is not the most economical angle—on the supposition that $b$ is constant—for this truss. It is hardly probable that eq. 14 would change this conclusion, which is different from that often given in text books.

103. *Applications.*—In the Triangular *Through Truss*, Fig. 7, it was assumed

that the unit strains are constant for all members but the three braces in the half truss, hence $m=o$ except for braces 1, 3, 5.

From the table art. 42 we find

Weights of chords $= W_c = 74,$
" vertical members $= W_v = 14,$
" ties and counters $= W_t = 29,$
" 3 braces $= W_b = 46,$

Now, $i=30°46'$ $\therefore$ $cos.2i=.477$; and 3 braces, $\frac{l}{d}=30$; also in eq. 13 "hinged ends," &c., $\frac{l^2}{r^2}=\frac{4}{9000}\left(\frac{l}{d}\right)^2$ and $\frac{h^2}{l^2}=.738$. Hence for the 3 braces, $m=\frac{3}{4}+\frac{4}{4}=1$;

$$\Sigma w\,(\cos.\,2i+\frac{h^2}{l^2}m)=46879(.477+.738)$$
$$=56,958$$

Also

$$W_t \cos.\,2i=29580\times.477=14110$$

For the vertical members $\cos.\,2i=1$

Now summing the results, we should *have* according to the rule of art. 98,

$$W_c = W_v + W_t \cos.2i + W_b\left(\cos.2i + \frac{h^2}{l^2}m\right)$$

But the numerical values give,

$$W_c = 74947 < 14336 + 14110 + 56958 = 85404.$$

104. With a less height $W_c$ would be larger. Let us then try $h=27$ ft.

It is far more direct now to compute the new weights of chords and web from eqs. (11) and (12).

Thus in eq. (11) making $\triangle h = -1$, we find the new weight of the chords to be,

$$W'_c = 74947 \tfrac{28}{27} = 77723.$$

Similarly, the new weight of vertical members is,

$$W'_v = 14336 \tfrac{27}{28} = 13824.$$

From eq. (12) making $\frac{b}{b'}=1$, we get the new weight of ties and counters,

$$W'_t = 29580 \left(\frac{31.7}{32.6}\right)^2 . \tfrac{28}{27} = 29000.$$

And from the same eq. (12) we find the new weight of braces 1, 3 and 5 to be

$$W'_b = 46880 \left(\frac{31.7}{32.6}\right)^2 . \frac{28}{27} . \frac{b}{b'} = 44455.$$

The braces were assumed, as before, of $13\frac{1}{16}''$ diameter. The new length of a brace is 31′.7; hence

$$\left(\frac{l}{d}\right)=29 \text{ and } \frac{b}{b'}=\frac{393}{406}.$$

We find, *cos.* 2 $i$=.448.

Apply eq. (14) again to the new weights:

$$W'_c = 77723 < 13824 + 29000 \times .448 + 44455(.448 + .706) = 78117$$

The truss may be one or two-tenths of a foot lower then for economy. The amount saved though is not worth the computing, for in the change from a height of 28′ to one of 27′ the amount saved is only 740 lbs. as we find from the above.

The time may more profitably be spent in ascertaining the best diameters of compression members for economy, regard being had to the castings and pins at the same time.

105. For the Whipple through truss, *Fig. 9,* we get from the table, art. 85,

$$W_c = 76208$$

Tie $\overline{Bc} \times \cos.\ 2i = 6064 \times .477 = 2892$

(Other ties and counters) $\times \cos.\ 2i' = 36000 \times -.174 = -6264$

$$\therefore\ \Sigma w_t \cos.\ 2i = -3372$$

Regard $b$ as constant for posts $\overline{Ff}$, $\overline{Gg}$,

$$\because\ W_v = \overline{Bb} + \overline{Ff} + \overline{Gg} = 7193$$

Also $W_p = \overline{Cc} + \overline{Dd} + \overline{Ee} = 17106$

For these posts $\frac{l}{d} = 34$, and for a "one pin end," $c\frac{l^2}{r^2} = \frac{1}{3000}\frac{l^2}{d^2}$ and $W_p(\cos.\ 2i + m) = 17106\ (1 + \frac{3}{4}\frac{4}{4} + \frac{1}{1}\frac{0}{8}) = 34469$

For the brace $Ba$, we have as before,

$$W_b\left(\cos.\ 2i + \left(\frac{h}{l}\right)^2 m\right) = 20495(.477 + .738) = 24901$$

Now from art. 98 we should have for economy

$$W_c = \Sigma w_t \cos.\ 2i + W_v + W_p(1 + m') + W_b\left(\cos.\ 2i + \left(\frac{h}{l}\right)^2 m\right)$$

whereas we find,

$$W_c = 76208 > -3372 + 7193 + 34469 + 24901 = 63191$$

$W_c$ is too great, hence the height should be greater. A height of 29 feet may now be tried.

The influence of the diagonals is very small in our equation, since *tan* $dBb =$ 50° or nearly 45° (art. 92). In a similar manner it is found that the height of the Whipple Deck Truss should be increased to 29 feet.

106. The Pratt Truss, through bridge, was next computed, for same span, loads, No. panels and height (28′) as before. The strains have already been given. The end brace was inclined, as in Fig. 9. Using the lettering of that Fig. the posts $\overline{Cc}$, $\overline{Dd}$ were given a diameter of 12″; the other of 8″ to 10″, the end brace and chord as before.

On making out a "Bill of Materials," the total weight of bridge was found to be 333,800 pounds.

From the table of weights,

$$W_c = 75{,}095 \text{ pounds.}$$

Ties and counters $= W_t = 36914$,

also *cos.* $2i = .477$

$W_v = \overline{Bb} + \overline{Ee} + \overline{Ff} + \overline{Gg} = 18654$
($b$ constant)

End brace, $W_b = 20495$ (as before)

Posts $\left\{ \begin{matrix} Cc \\ Dd \end{matrix} \right\} = W_p = 18174.$

For these posts $\frac{l}{d} = 28$ and,

$W_p (\cos. 2i + m') = 18174(1 + \frac{7}{17} + \frac{3}{4})$
$= 33374.$

Then for economy we should have

$W_c = W_t \cos. 2i + W_v + W_b \left(\cos. 2i + \frac{h^2}{l^2} m\right) + W_p (1 + m')$

Actually we find,

$W_c = 75095 < 17608 + 18654 + 24901 + 33374 = 94537$

The height is too great, $h = 26$ may be tried.

107. On computing, by eqs. (11) and (12), the new weights of the Whipple Truss for a height of 29′ and of the Pratt Truss for a height of 26′, we find a saving in the former of 544 pounds, and in the latter of 714 pounds over the

weights of the respective tr
high.

From eq. (14) we also asce
the Whipple Truss, *for the*
*taken*, might have a height o
say over 29′ with economy.
nas within a tenth of a foot of
economical height for the d
given.

108. Let us now compare th
of the three trusses examine
most economical heights. As
eters of the columns are uncha
same number of pounds of iro
ings, &c., was added as bef
section of the vertical post
triangular truss was taken at 4
inches (see art. 87).

The trusses are all of 200′ s
12 panels. Assumed dead loa
lbs ; live load 2,000 lbs. per f
two 60,000 lbs. weights, not
50′ apart, so placed as to give
strains in chords and web. Th
for the diameters of columns, s
unit &c., given, are of the mos

ical heights ; all of them being through bridges with leaning end posts. The following is the comparison of weights:

| Truss. | Fig. | Height. | Weight in lbs. |
|---|---|---|---|
| Triangular.... | 7 | 27 | 324909 |
| Whipple..... | 9 | 29 | 325390 |
| Pratt..... | 5 | 26 | 333086 |

The comparison is thus most favorable to the Triangular, next to the Whipple, and least to the Pratt Truss, for the panel length &c., taken. Practically, the first two have the same weight.

109. An increase of the diameters of those columns that admit of it, would probably benefit the triangular most. Thus some of the interior posts of the Pratt or Whipple Trusses admit of little or no increase in diameter for a proper thickness of metal, whereas the main braces of the triangular do admit of it. With diameters of 15″ for upper chords and braces, the triangular may give the least weight; supposing the diameters of the upper chords of the other

trusses to be 15″ also, the posts being enlarged where possible. On the contrary the workmanship towards the center of the space probably costs more for the triangular than for the others.

The heavy competition in this country has been productive of economy in material and workmanship, in bridge building, and the "bids" on the same design, often give the best comparisons between trusses of different types and details.

Each design has its advantages and disadvantages, and as a consequence its advocates and opposers.

A proper study of the details of trusses now before the country is then imperative.

110. It is interesting to ascertain what inclinations of ties and braces will make the *web material* a minimum. Thus let Fig. 12 represent a panel of Fig. 11. Put AB$=l$, AC$=l_1$, BC$=l_2$, AD$=x$, DC$=h$; the dimensions being in inches.

$w''$ = weight of 1 cu. in. of tie BC in lbs.

$w'$ = weight of 1 cu. in. of post AC in lbs.

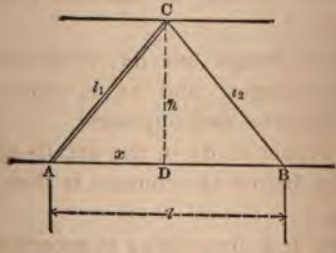
$c''$ = cost per pound of tie in cents.
$c'$ = cost per pound of post in cents.
$b = 7500\,(1+\theta)$ = strain per sq. in. for tie.

$$b' = \frac{38500\,(1+\theta)}{4 + \frac{l_1}{10d} + \frac{4cl_1^2}{r^2} + \frac{cl_1^3}{10dr^2}} = \text{safe strain}$$

per square inch for post,

as given by eq. (8), art. 53.

FIG. 12.




We find, S being the shear on the panel,

Strain on CB $= S\frac{l_2}{h}$; cost CB $= \frac{Sl_2}{hb'} l_2 w'' c''$.

Strain on CA $= S\frac{l_1}{h}$; cost CA $= \frac{Sl_1}{hb'} l_1 w' c'$.

Substituting for $b$ and $b'$ their values, we

have as the total cost of tie CB and post CA,

$$\frac{S}{100\ (1+\theta)h}\left\{\frac{l^2_2 w''c''}{75}+\frac{l^2_1(4+\frac{l_1}{10d}+4c\frac{l^2_1}{r^2}+\frac{cl^2_1}{10dr^2})\ w'c'}{385}\right\}.$$

Placing $l_1=\sqrt{h^2+x^2}$; $l^2_2=h^2+(l-x)^2$; differentiating with respect to $x$ and placing the result $=o$, we find that for *the least cost of tie and post* (on replacing $\sqrt{h^2+x^2}$ by $l_1$),

$$x=\frac{308\ w''c''l}{308w''c''+30w'c'(8+\frac{3l_1}{10d}+16c\frac{l^2_1}{r^2}+\frac{c}{2}\frac{l^2_1}{dr^2}}$$

*Examples.*—1. Let $w''c''=w'c'$, $\frac{l_1}{d}=30$, and for a hollow cylindrical post hinged at both ends $c=\frac{1}{18000}$, $r^2=\frac{d^2}{8}=\frac{l^2_1}{7200}$, whence $x=.26\ l$.

If $d$ is given, it is evident that $h$ has only one value corresponding to $x=.26\ l$ to be found from the equation $\frac{l_1}{d}=30=\frac{\sqrt{h^2+x^2}}{d}$ $\therefore\ h=\sqrt{(30d)^2-x^2}$.

2. Similarly we find for $\frac{l_1}{d}=20\ x=.36l$; and for $\frac{l_1}{d}=40$, $x=.18l$.

111. If for $b^{1\prime}$ we write Rankine's formula with a constant factor of safety 5,

$$\therefore\ b^1=\tfrac{1}{5}\cdot\frac{38500}{1+c\frac{l^2}{r^2}}$$

and proceed as before to deduce a formula, &c., we shall find by it that for

$$\frac{l_1}{d}=20,\ x=.36l$$
$$30,\ x=.3\ l$$
$$40,\ x=.24l$$

If in a Pratt truss (Fig. 5) of 200′ span, the *posts* as well as the ties are inclined so that $x=\frac{1}{3}l$, the web (neglecting the counter braces) weighs a few thousand pounds less than with vertical posts; the posts regarded as hinged at both ends in both cases. Using the value of $b'$ in the *previous* article, for $\frac{l_1}{d}=30$ as an

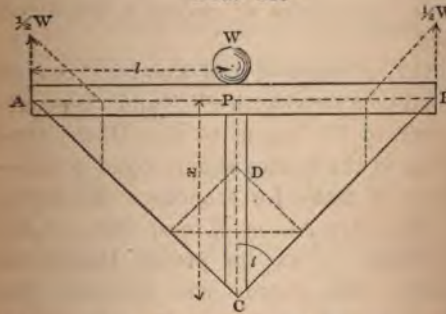
average, we found $x=\frac{1}{4}l$ for greatest economy. This is the value adopted in the Post truss and is, theoretically, correct for the above value of $b'$, which is agreeable to practice as before mentioned. The *economy* of the square joint, however, due both to less workmanship as well as the use of a formula for posts with "flat ends" or "one pin ends" eliminates all saving in this direction.

112. If the post is of wood $w'c'$ is very small compared with $w''c''$ and $x$ is nearly equal to $l$. Hence in the Howe type (Fig. 6) the braces should be of wood—never of iron—for economy. Similarly, if the post AC is of cast iron, $x$ approaches $\frac{1}{2}\,l$ as its proper theoretical value. The chords will influence the above results very slightly for usual diameters of upper chord.

113. For deck bridges and the triangular through truss, the shear on the post is greater than on the tie, and the post should be more nearly vertical. *This* supposition is easily included in the *formula*.

114. *Most Economical Depth for a Fink Element.*—Let a weight W act at P, Fig. 13. Call the constant length, BP=AP=$l$; the variable height of post PC=$x$; the strain per square inch on ties AC, BC, = T, on chord AB=$b'$, on post PC=$b$. The weight W is directly supported by the post PC.

FIG. 13.




Decomposing W=DC at C, the strain on AC or BC is $\frac{1}{2}$W *sec.* $i=\frac{W}{2}\frac{\sqrt{x^2+l^2}}{x}$.

This strain is in equilibrium at A or B with the reaction $\frac{1}{2}$W and the chord resistance $\frac{1}{2}W$ *tan* $i=\frac{W}{2}\frac{l}{x}$.

On dividing the strain on each member by its strain per square inch and multiplying by the length of the member in inches, we get its volume. Thus the total volume of AC+BC+PC+AB is

$$W\left(\frac{x^2+l^2}{Tx}+\frac{x}{b}+\frac{l^2}{xb'}\right), \textit{ to be a min.}$$

Now T is constant; also $b'$, since $l$ is constant, but $b$ varies with $x$.

115. *Regarding $b$ as constant;* on differentiating, &c., we readily find, *for a min. vol.*

$$\frac{l^2}{x^2}\left(\frac{1}{T}-\frac{1}{b'}\right)=\frac{1}{T}+\frac{1}{b}$$

$$\therefore \textit{ tan. } i=\frac{l}{x}=\sqrt{\frac{(b+T)b'}{(b'+T)b}}$$

Thus let $T=b'=10000$,

$b=10000$, $i=45°$
$b=\ 7000$, $i=47°47'$
$b=\ 6000$, $i=49°\ 3'$
$b=\ 5000$, $i=50°47'$
$b=\ 4000$, $i=52°55'$, &c.

116. Regarding $b=\frac{38500(1+\theta)}{\left(4+\frac{1}{10}\frac{x}{d}\right)\left(1+c\frac{x^2}{r^2}\right)}$

as variable we find,

$$\text{tan. } i=\frac{l}{x}=$$

$$\sqrt{\frac{b'\mathrm{T}}{b'+\mathrm{T}}\left\{\frac{4+\frac{x}{5d}+\frac{12cx^2}{r^2}+\frac{4}{10}\frac{cx^2}{dr^2}}{38500(1+\theta)}+\frac{1}{\mathrm{T}}\right\}}$$

For a given $\frac{x}{d}$ we can of course find *tan. i*; but generally $d$ is given and we can not know $\frac{x}{d}$ until $x$ is found. Hence, given $d$, we cannot determine *tan i*, except by a series of approximations.

But as in the case of beam trusses, having assumed $x$ and $d$, unless the preceding equality holds, the most economical depth has not been chosen, and the formula will indicate whether $x$ is too small or the reverse.

*Examples.*—Let, $b' = \mathrm{T} = 10000$, and let PC be a hollow cylindrical column

$\therefore \frac{x^2}{r^2}=\frac{8x^2}{d^2}$; $\frac{x^3}{dr^2}=\frac{8x^3}{d^3}$. Also place $c=\frac{1}{24000}$ and $\theta=\frac{1}{3}$.

Then for,

$\frac{x}{d}=20$, *tan.* $i=1.237$ $\therefore$ $i=51°\ 3'$

$\frac{x}{d}=30$, *tan.* $i=1.473$ $\therefore$ $i=55°50'$

$\frac{x}{d}=40$, *tan.* $i=1.761$ $\therefore$ $i=60°25'$

Thus somewhat shorter posts are required than when $b$ is taken constant.

117. Let us now investigate a *Fink truss* (deck bridge) Fig. 14 for maximum strains and minimum material. Assume as before a 200′ span, but divide it into 16 panels of 12½′ each. As before, let the weight of bridge=336000 lbs., or 10500 lbs. per panel on one truss; the car load, uniformly distributed, 1000 lbs. per foot or 12500 per panel for one truss, and the locomotive excess, two weights, 30000 pounds each for one truss and 50′ apart, to be so placed as to give maximum strains on chords, posts or chain *system.*

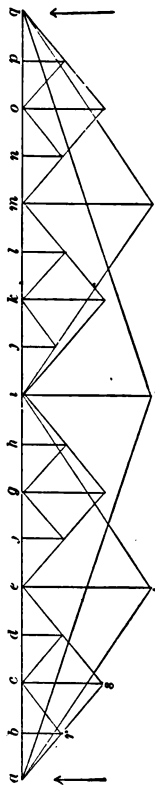



FIG. 14.

Each 30,000 pounds rests on 3 drivers for one truss, 6′ apart or a total wheel base of 12′ ∴ there is 10,000 pounds on each driver. *Hence when the center driver is at any post as c, the adjoining posts bear* $\frac{6}{12.5}$ 10,000=4800 *lbs., and the post c therefore* 30,000−9600=20400 *directly.*

118. If a weight is placed anywhere on $\overline{ac}$, since the element *arc* acts independently, *the reactions at a* and *c are determined by the law of the lever.* Similarly for the systems *ase*, *ati*, and *auq*; for the posts at the end of the system act as abutments to the system considered, and the reactions can only be determined by the simple law of the lever, irrespective of the pattern of the chain system used.

119. It follows, therefore, that the max. strains on posts *b*, *d*, *f* . . . . =1 panel dead and car load, (23000) + 20400 of loc. excess (center driver bearing on post)=43400 pounds.

The dead load is really less as the

chains, *at*, *au* . . . only rest on *cs*, *ct* . . . ordinarily, but the section of the post would hardly be taken less than this strain gives, owing to oscillation of engine sometimes increasing the reaction at *b*, *d* . . . due to engine weight. But for posts *b* . . . put $\theta = \frac{5000}{43400} = .13$.

Next, let center driver bear at *c*.

The post *c* bears directly 23000 lbs. car and dead load + 20400 loc. excess; also 23000 + 4800 transferred from *b* and *d*, making in all 71200 $\therefore \theta = \frac{21000}{71200} = .3$.

The post *e*, bears directly $43400 + \frac{1}{2}$ car and dead load at *b*, *c*, *d* and *f*, *g*, *h*, $(69000) + (2.\frac{3}{4}.4800 + \frac{1}{4}.4800)$ loc. weights borne at *d*, *f* and *h* = 120800 lbs. *if* center driver is at *e*; but with locomotives at *c* and *g*, post *e* sustains $\frac{1}{2}$ load on *a* *i* = 122000 lbs. which is therefore its max. strain; and $\theta = \frac{4^2}{1^2 2^2} = .34$.

If locomotives are supposed at *d* and *h*, the reaction at *e* due to them is $(\frac{3}{4} + \frac{1}{4})$ 30000 = 30000 as in the preceding case.

*Lastly to find* the max. strain borne

by post $i$. It bears 8 panels, car and dead load (184000 lbs). With engines at $g$ and $k$, by art. 118, post $i$ bears $\frac{3}{4}60000$ loc. load. With engines at $h$ and $l$, $i$ sustains $\frac{7}{8}30000+\frac{5}{8}30000=\frac{3}{2}30000$ as before; but with engines at $i$ and $m$, $i$ sustains $\frac{1}{2}30000+2\frac{1}{8}4800+20400=43800$ or less than the 45000 before.

120. From the above we see that when one locomotive only can get on the system, it must be placed over the central post of that system to find its max. strains; when two locomotives can bear on one system they must be placed either side of the central post.

The above strains are entered in the following table. The max. strains on the ties at the foot of the posts are found by multiplying $\frac{1}{2}$ the max. strains on posts by *sec.* $i$, $i$ being the inclination of the tie to the vertical.

The lengths and diameters of posts are assumed as in the table.

It was not considered judicious to make the center post $i$ longer than 30 *diameters*, though for theoretical economy it should be much longer.

| Piece. | $d$ | $\frac{l}{d}$ | th | Strain. | $\theta$. | $b$. | Area. | Length. | No. | $k$. | Weight. | Totals. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | " | | " | | | | □" | ' | | | lbs. | |
| Post *b*..... | 6 | 20 | $\frac{5}{16}$ | 43400 | .13 | 6400 | 6.8 | 10 | 16 | $\frac{10}{8}$ | 3627 | |
| *c*..... | 11 | 22 | $\frac{5}{16}$ | 71200 | .30 | 6950 | 10.3 | 20 | 8 | " | 5493 | |
| *e*..... | $13\frac{1}{8}$ | 30 | $\frac{1}{2}$ | 122000 | .34 | 5670 | 21.5 | $\frac{100}{3}$ | 4 | " | 9555 | |
| *i*..... | $13\frac{1}{8}$ | 30 | 1 | 229000 | .37 | 5800 | 39.5 | $\frac{100}{3}$ | 2 | " | 8778 | 27453 |
| Tie *ar*..... | | | | 34720 | .13 | 8470 | 4.1 | 16 | 32 | " | 6997 | |
| *as*..... | | | | 56960 | .30 | 9750 | 5.8 | 32 | 16 | " | 9898 | |
| *at*..... | | | | 109983 | .34 | 10050 | 10.9 | 60.1 | 8 | " | 17469 | |
| *au*..... | | | | 362049 | .37 | 10420 | 34.7 | 105.4 | 4 | " | 48765 | 83129 |
| ard *aq*..... | $13\frac{1}{8}$ | 11.3 | $1\frac{3}{16}$ | 485625 | .39 | 10140 | 47.9 | 200 | 2 | " | 63867 | 63867 |

121. *Chord Strains.*—As in art. 114, to find the chord strain due to any element we multiply $\frac{1}{2}$ weight at foot of post by *tan. i.*

Thus for the uniformly distributed car and dead load of 23000 lbs. per panel, post *b* bears 23000 lbs; post *c*, 46000; post *e*, 92000 and post *i*, 184000 lbs. Similarly for similar posts so that the strain on *a q*, for uniform load is the same throughout and equals

$$\tfrac{1}{2}(184000\times 3+92000\times 1\tfrac{1}{2}+46000\times\tfrac{3}{2}\tfrac{5}{0}+23000\times 1\tfrac{1}{4})=388125 \text{ lbs.}$$

122. Next consider the locomotive excesses, 50′ apart, consisting of 30000 lbs. each on 3 drivers. With center drivers at *g* and *k*, these posts support directly and indirectly 25200, the adjacent posts 4800 lbs each (art. 117); *e* and *m*, will bear 15000, and *i* 45000 lbs. applying the simple law of the lever to determine these reactions. This gives as the total strain on the parts *ei* or *im*, due to loc. excess

$$\tfrac{1}{2}(45000\times 3+15000\times 1\tfrac{1}{2}+25200\tfrac{3}{3}\tfrac{5}{0}+4800\tfrac{3}{2}\tfrac{5}{0})=97500 \text{ lbs.}$$

Similarly, for engines at $e$ and $i$, the part $\overline{ci}$ experiences a strain of 92400 lbs. which differs but little from the preceding; hence I have regarded the chord $\overline{aq}$ as strained throughout by 97500+388125 =485625 lbs. as entered in the table.

With engines at $c$ and $g$, the chord strain on $\overline{ai}$ due to loc. excess is 86250 lbs.—less than in preceding cases.

123. The trusses were assumed 14′ from center to center; floor beams being 15.5′ long and 24″ deep; the web, $\frac{1}{2}$″ thick. The loss in the rivet holes is assumed equal in effect to the resistance afforded by the web &c. The floor beam max. live load is 63880 lbs. (see art. 15), to which add 6738 lbs. dead load. The moment at center is thus, $35309\times54''=fda=7500\times24\times10.6$. The section of a floor beam is thus, 28.5 sq. in. and its weight 1472 lbs. Similarly the stringers of wood, each $16''\times6.6$ (see Fig. 9) or of iron I beams, 16″ deep, weigh about 213 lbs. per foot. The transverse bracing was put at 11400 lbs. as for the Whipple truss, the rails and

cross ties as before. The "Whipple" deck truss, (art. 91) with which this one will be compared was subjected to as near the same conditions as possible, except that the panel length of the former was taken at $16\frac{2}{3}$ feet, whereas a different panel length might be more economical. The same percentages for castings, bolts, &c., was added to both.

The following is the

BILL OF MATERIALS.

Fink Deck Bridge, 200′ span, 16′ panels.

| | lbs. |
|---|---|
| Ties | 83129 |
| 15 p. c. | 12469 |
| Chord and posts | 91320 |
| 20 p. c. | 18264 |
| Lateral tie rods and struts | 11400 |
| 17 Floor beams, 24″ deep | 25000 |
| Wooden stingers | 42600 |
| Rails, cross ties, &c. | 33200 |
| Total weight of bridge | 317382 |
| Assumed weight | 336000 |
| Assumed weight too great by | 18,618 |

124. Let us now ascertain if each element of the truss has its most economical *depth*.

For the element *arc*, we must substitute in the value of *tan. i* (art. 116), T= 8500, $b'$=10200, $\frac{x}{d}$=20 and $\theta$=.13, as found from the table; whence we find that for the most economical depth $\frac{l}{x}$=1.3 ∴ for $l$=12.5, $x$=$\overline{br}$=9.6 feet, As we assumed $\overline{br}$=10, the result is almost exact; in fact considering the thickness of chord, it is practically exact.

Similarly for the element *ase*: T= 9800, $b'$=10200, $\theta$=.3, $\frac{x}{d}$=22 whence *tan* $i=\frac{l}{x}=\frac{4}{3}$ ∴ for $l$=25, $x$=$\overline{cs}$=18.8 feet.

This value differs only 1.2 feet from the 20 feet assumed, or really only .6 foot say, considering the thickness of chord. The depth is very slightly too great.

For the element *ati*, T=10050 $b'$= 10140, $\frac{x}{d}$=30 and $\theta$=.34, whence (see 2nd example, art. 116) $\frac{l}{x}$=1.473 ∴ for $l$

$=50$, $x=et=33.9$, we assumed 3
Practically then, the most economi
depths have been chosen for all the
ments excepting *auq*, which is necessa
circumscribed in depth.

125. *The formula of art.* 98 appl
directly to a Fink element, Fig. 13, si
the chord strain varies directly as *ta*
or *as the depth*, and the shearing for
$\frac{1}{2}$ W, is the same on the ties of Fig.
for any depth; these being the o
requirements of the formula.

For a Fink element, Fig. 13, form
(14), art. 98, takes now the followi
shape,

$$W_c = W_t \cos. 2i + W_p (1+m);$$

in which

$W_c$ = Weight of chord A B

$W_t$ = Weight of ties AC + CB.

and

$W_p$ = Weight of post PC.

In the value of $m$, for hollow cylind
cal posts, hinged at one end,

$$c\frac{l^2}{r^2} = \frac{1}{3000}\left(\frac{l}{d}\right)^2.$$

Now for the element *ati* Fig. 14, $i = 56°19'$, $\cos. 2i = -.385$.

From the table art. 120 we get $4\ W_t = 17469$, $4\ W_p = 9555$; and computing $W_c$ we find $4\ W_c = (61000 \times 6 \div 10140)^{\frac{100}{3}} = 12030$. Also for $\frac{x}{d} = 30$, $m = \frac{3}{4} + \frac{1}{39}$; whence

$$4\ W_c = 12030 > 17469 \times -.385 + 9555\,(1 + \tfrac{3}{4} + \tfrac{1}{39}) = 11235$$

The chord weight is very slightly too great, which indicates that the depth is too small for the most perfect economy; the same conclusion previously arrived at.

*On comparing now the weight of the Fink with that of the Whipple deck bridge*, (*art.* 91), *we see that the Fink is lighter by* 10,072 *lbs.*

126. *Fink Through Bridge.*—If we draw a line *tu* (Fig. 14) parallel to chord and drop "suspenders" from the foot of posts, as *r*, *s*, . . . to hold up the roadway *tu*, and also add vertical posts at *a* and *q*, the depth of the truss, we have an outline drawing of the Fink through bridge. Call the points of the roadway

vertically under $a$, $b$, $c$, . . . . respectively $a'$, $b'$, $c'$ . . . .; and consider the element $arc$ conjointly with the suspender $rb'$ for economy. Call $br=x$, $bb'=h$ $\therefore$ $rb'=(h-x)$. As the post $br$ only supports one panel upper chord, &c., its section will practically be taken much larger than the 2500 pounds about of dead load requires. Hence we can regard its section=S *constant* as $br$ varies in length.

As in art. 114, call the strain per square inch on ties ($ar$, $rc$, $rb'$), T; on chord, $b'$. Then we have as in art. 114 the total volume of $ar$, $rc$, $b'r$, $ac$ and $br$ (calling W=load on suspender $b'r$),

$$W\left(\frac{x^2+l^2}{Tx}+\frac{h-x}{T}+\frac{l^2}{xb'}\right)+Sx=a\ min.$$

whence,

$$\frac{l}{x}=\frac{bc}{br}=\sqrt{\frac{b'TS}{W(b'+T)}}$$

Now putting S=4.5, $b'$=10000, T=8500 W=43400, it follows that for economy that $x=l\div.685$. Thus if $l=12.5$, $x=br=18$. For S=10, $x=12.5$ feet, &c.

The first values (nearly) are taken from the following table, and the results are thus correct for the element *arc* : but in the element *ase*, the cross section of *cs* must not be assumed constant for different depths. If it were, then it follows that for $b'=10{,}000$, $T=9700$, $W=71200$ and $S=8$ that $l=\frac{3}{4}x$ $\therefore$ $x=33.3$ feet$=cs$. For $S=14$ (about), $x=l$.

It is easy to deduce a formula regarding $b$ for the post *sc* as varying, but it is perhaps simpler to determine the proper value for *sc* by trial. From the formula above we see that as S diminishes, that the angle between the ties becomes less.

127. Let us assume as before that the ties *ar* and *as* are equally inclined, but place their inclination now at 45°, the depth of truss, no. panels, &c., being assumed as before.

The maximum strains on ties and top chord are determined as before, since they depend only upon the load borne at the foot of each post, whether that load is communicated by posts or suspenders or both.

The chord strain due to uniform car and dead load

$$=\tfrac{1}{2}(184000\times 3+92000\times 1\tfrac{1}{2}+46000+23000)=379500;$$

and that due to loc. excess placed at $g'$ and $k'$

$$=\tfrac{1}{2}(45000\times 3+15000\times 1\tfrac{1}{2}+30000)=93750$$

The sum of the two is entered in the following table. With engine at $b'$, post $c$ bears about 41000 pounds. The posts $e$ and $i$ bear, one panel of car load (12500) + one panel roadway (3535), or 16035 pounds *less than before*, giving the max. loads ever borne by

Post $e$, $122000-16035=105965$
" $i$, $229000-16035=212965$

The dead loads carried by these posts are 17465, 38465 and 80465 so that $\theta$ has the respective values, 42, 36, 37.

The suspenders bear 32900 lbs. live load ($=20400+12500$) and 3100 lbs. *roadway*: in all 36000 lbs.

| Piece. | $d$ | $\frac{l}{d}$ | th | Strain. | $\theta$. | $b$ | Area. | Length. | No. | $k$. | Weight. | Totals. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ″ | | ″ | | | | □″ | ′ | | | lbs. | |
| Chord..... | $13\frac{1}{8}$ | 11.3 | $1\frac{8}{16}$ | 473250 | .39 | 10140 | 46.7 | 200 | 2 | $\frac{1}{10}$ | 62267 | 62267 |
| Post *br*..... | 5 | | $\frac{1}{4}$ | 2500 | 1. | | 3.6 | 12.5 | 16 | " | 2400 | |
| *cs*..... | $7\frac{7}{8}$ | 37 | $\frac{6}{16}$ | 41000 | .42 | 4880 | 8.4 | 25 | 8 | " | 5600 | |
| *et*..... | $13\frac{1}{4}$ | 30 | $\frac{1}{2}$ | 105965 | .36 | 5750 | 18.4 | $1\frac{0}{10}$ | 4 | " | 8180 | |
| *iu*..... | $13\frac{1}{8}$ | 30 | $\frac{11}{16}$ | 212965 | .37 | 5800 | 35.5 | $1\frac{0}{10}$ | 2 | " | 7889 | 24069 |
| Suspenders.. | | | | 36000 | .13 | 8470 | 4.3 | 20.8 | 16 | " | 4770 | |
| " .. | | | | 36000 | .13 | 8470 | 4 3 | 8.3 | 8 | " | 952 | 5722 |
| Tie *ar*..... | $\frac{43400}{2}$ | $\cdot\frac{35.4}{25}$ | = | 30727 | .13 | 8470 | 3.6 | 17.7 | 32 | " | 6797 | |
| *as*..... | $\frac{71200}{2}$ | $\cdot\frac{35.4}{35}$ | = | 50410 | .3 | 9750 | 5.2 | 35.4 | 16 | " | 9818 | |
| *at*..... | $\frac{122000}{2}$ | $\cdot\frac{60.1}{33.3}$ | = | 109983 | .34 | 10050 | 10.9 | 60 1 | 8 | " | 17469 | |
| *au*..... | $\frac{229000}{2}$ | $\cdot\frac{105.4}{33.3}$ | = | 362049 | .37 | 10420 | 34.7 | 105.4 | 4 | " | 48765 | 82849 |

128. With trusses 16′ apart, center to center, the iron floor beams, 26″ deep are estimated to weigh 1866 lbs. weight per panel; stringers and track as before 4738 lbs. We now form the following:

BILL OF MATERIALS.

Fink through bridge, 200′ span, 16 panels, 33′.3 deep.

| | lbs. |
|---|---|
| Chain system and suspenders | 88571 |
| 15 p. c. for bolts, nuts, eyes and pins | 13286 |
| Chord and posts | 86336 |
| 20 p. c. for castings | 17267 |
| Floor beam loops | 5000 |
| Lateral rods, struts and portals | 15000 |
| 15 floor beams (26″ deep) | 27990 |
| Stringers (of wood) | 42600 |
| Rails, cross ties, &c | 33200 |
| Total weight | 329250 |
| Assumed weight | 336000 |

The lateral struts and portals were increased over previous trusses examined by 4600 lbs. on account of the greater depth of this truss. The roadway for greater stability, should be formed of *closely* spaced cross bearers, extending

from truss to truss, but we have *estimated* as aboye. If we subtract the weight of portals, say 5000 lbs. we get the weight of bridge for calculation 324,250 lbs.

The four-end posts or "pier towers," are 33.3 feet high and for 30 diameters weigh in all 17111 lbs. since they sustain a max. load of 226500, when train extends from farthest abutment to nearest panel. These pier towers should be given a broader base for equal stability with other trusses, and hence should weigh more than the above. Putting them at 17111 the total weight of bridge and towers is 346,361 lbs. which is more than for the Pratt, Whipple or Triangular, previously examined.

129. To ascertain if the most economical depth has been chosen, let us keep the inclination of the ties, inclined 45,° at that angle. Then for a change of height $\triangle h$, only the suspenders, end posts and weights, due to systems *auq* and *ati*, need be examined. Call $w_s$ = weight of suspenders of height $rb' = 20.8 = h_1$ then if the height of the truss $h = 33.3$ is

increased by $\triangle h$, *the new weight of suspenders* is

$$w_s \frac{h_1 + \triangle h}{h_1} = w_s + w_s \frac{\triangle h}{h_1}$$

Similarly the new weight of the other suspenders whose height $= h_2 = 8.3$ is

$$w_s' + w_s' \frac{\triangle h}{h_2}$$

If these expressions are added to the value of $F(h + \triangle h)$ in art. 98, and the subsequent reductions made as in that article, (the transformations the above terms undergo are very easily traced), we find in place of *eq.* (14) that for the most economical height,

$$W_c = w_s \frac{h}{h_1} + w_s' \frac{h}{h_2} + \Sigma w(\cos.\ 2i + \frac{h^2}{l^2} m)$$

130. In this formula $w_c$ = weight of chord due to variable elements *ati* and *auq* = 55733 lbs. The posts *et*, *ia* and end posts being all 30 diameters long and vertical, $\cos. 2i = 1$, $\frac{h^2}{l^2} = 1$, $m = \frac{3}{4} + \frac{1}{3\,9}$, and their weight $W_p = 16069 + 17111 = 33180$ *lbs.*

The weight of ties, *at*, *ti*, . . . , inclined at 56°19 to the vertical is $w_t = 17469$. For them *cos.* $2i = -.385$. The w't. of *au uq* $= w_t' = 48765$. For them *cos.* $2i = -.6$. Also $w_s = 4770$ and $w_s' = 952$.

Now if the most economical height has been chosen we should have

$$W_c = w_s \frac{h}{h_1} + w_s' \frac{h}{h_2} + W_t \times (-.385) + W_t'(-.6) + W_p (1+m).$$

Actually we have,

$$W_c = 55732 > 11250 - 35985 + 62714 = 37979,$$

The chords being the greater, the depth ($33\frac{1}{3}'$) is too small for theoretical economy; but it would hardly be prudent to increase it.

131. If however we take the pier towers at 27000, the right member equals 57096, which nearly equals $W_c$.

132. The Fink truss is better adapted for a deck than a through bridge, and possesses one advantage for either form over all others—"compensation" under live loads or changes of temperature;

each isosceles triangle or "element" being independent in its action of every other, there can be no loose counters causing distortion of the bridge as in some beam trusses. Its action in practice is said to be "perfect." It would seem that an increase of diameter of the heavy chord of the Fink would benefit it more than a similar operation would benefit the quadrangular trusses. A re-estimate can alone determine.

132. It may be remarked that the number of panels in a Fink is, from the peculiar design, some power of 2; 2, 4, 8, 16, 32, &c., thus fixing a panel's length. The Quadrangular trusses on the contrary can have any number of panels, thus giving it a greater adaptability to various spans. The Triangular truss with suspenders has necessarily an even number of panels. But with some of the center panels built on the quadrangular plan (as has been done), thus giving short posts, where material is most likely to be wasted, the adaptability to any n can be made equal to that of the *drang*ular trusses.

For very long and deep spans—300 to 600 feet and upwards—vertical posts throughout would seem to be desirable; for if inclined, the flexure from their own weight would be appreciable and might continually increase besides.

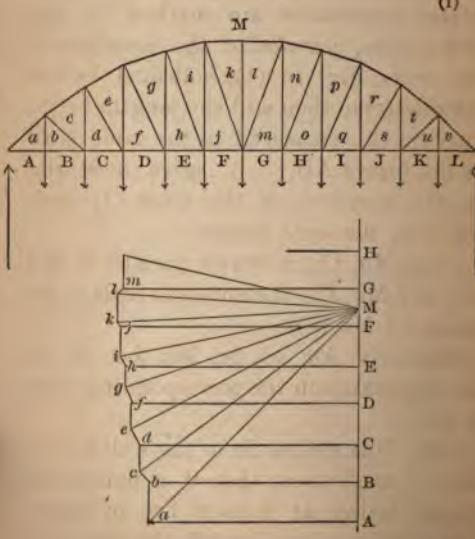
The posts, when the truss is high enough to admit of it, are often braced together between their ends, thus shortening practically their length as compared with their diameters, and adding materially to the stiffness of the system.

133. *Bow String Girder.*—This truss, depicted in Fig. 15, is assumed, as before, to have 200 feet span, divided into 12 panels, and to weigh 336000 lbs. (entire bridge). The center height is assumed at 30 feet. The counters are omitted in Fig. 15, as we shall assume them out of action when the bridge is loaded uniformly, which case will first be investigated.

134. Using Bow's ("Economics of Construction," &c.), notation, we denote any bridge member, in Fig. 15 (1), by the two letters placed either side of it,

thus the first vertical on the left is *ab*, the next *cd*, &c. Similarly the first bow piece on the left is *a*M, the next *c*M, &c., the first chord piece is *a*A, the next *b*B, &c. The same notation applies to the forces AM, AB, BC, &c.

FIG. 15.



135. Let the bridge be loaded with its own weight only, 14000 lbs. per panel on one truss; *i.e.*, AB=BC=CD=&c.= 14000 lbs. Then the reactions AM and LM are $5\frac{1}{2} \times 14000$.

Lay off, in Fig. 15 (2) the forces AB, BC, &c. vertically, also the reaction AM, and draw the lines, as per figure, whose extremities are marked by any two letters, *parallel* to the members of the truss, Fig. 15 (1), indicated by the same letters; then will the lengths of the lines in (2), measured to the same scale as the forces AB, . . . . give the strains on the members of the truss (1) indicated by the same letters.

Thus M*a*, (2), is drawn parallel to M*a*, (1), and M*a*, (2), measured to scale is the strain on M*a* (1).

Similarly A*a*, *ab*, *bc*, M*c*, &c., in (2) are the strains on the corresponding parts in (1).

136. This results from the well-known law of mechanics, that if a number of forces acting at a point are in equilibrium, then if we lay off the forces in

order, "the polygon should close." Also, having given, at any apex, the direction of one force, by following around the corresponding polygon we find the directions of the others. If the force, representing the stress on a member, is thus found to *act away from the apex*, the member is in tension, *if towards the apex* the member is in compression.

Thus at apex AM*a*, AM is given acting upwards: then in (2) following around the polygon AM*a*A in order, M*a* is found to act *towards*, and *a*A from the apex; *i.e.*, *a*M, (1), is in compression and *a*A, (1) in tension.

Be careful to note now that these same pieces *act* in an opposite direction at their other ends. Thus at apex AB*ab* (1), *a*A acts to the left, *being in tension;* then following around the corresponding force polygon (2) in the order A*ab*BA, we find *ab* and *b*B acting away from the apex, hence in tension. Next at apex M*abc* (1), *a*M*cba* (2) gives CM compression and *bc* tension. Similarly all the *web* members will be found in tension,

the bow in compression and the chord in tension.

We determine first the strains at a chord apex, to find the strain on the vertical, then go to the bow apex above it, where, the strains in *two pieces* only being unknown, can be readily found.

137. The strains were of course determined from a larger drawing, the truss being drawn to a scale of 10 feet= 1 inch, and the force diagram being drawn to a scale of 20000 lbs.=1 inch.

We thus find that the *dead load alone* strains the ties, *bc*, *de*, *fg*, *hi*, *jk*, 4700, 4000, 3300, 2900 and 1300 pounds respectively. The verticals thus carry the greater part of the weights. With engine at the foot of tie *ab*, its maximum strain is 45000 lbs.

It was not considered judicious to proportion the verticals (when acting as ties) for a less strain, as a very slight error in the length of a diagonal could cause the vertical (neglecting the counter, supposed loose however) to sustain the whole panel reaction of 45000 lbs.

138. For a uniform live load, the force diagram is similar to (2), and the bow and chord strains can be most conveniently obtained from it. With the locomotive excess placed so as to give max. chord strains, a new diagram would be required for every new position of the load however; and on that account it is simplest to use the principle of moments.

139. The maximum moment about an apex $n$ panels distant from the abutment is given by the eq., art. 39, by simply dividing by $h$, as is sufficiently evident.

$$\therefore \mathrm{M}_n = \left\{ \frac{\mathrm{P}l}{2}(\mathrm{N}-n) + \frac{\mathrm{E}l}{\mathrm{N}}(\mathrm{N} - \frac{c}{l} - n) \right\} n$$

In the case of the Bow String girder, the lever arms for panels A$a$, C$d$, D$f$, E$h$, F$j$ respectively are $ab$, $cd$, $ef$, $gh$ and $ij$ respectively. The lever arms for the arch panels M$a$, M$c$, M$e$, M$g$ . . . are the perpendiculars drawn from the apices A$b$, B$d$, C$f$, . . . . respectively to the chords of the arcs M$a$, M$c$ . . . .

*Substituting* now in the last eq.,

$N=12$, $l=\frac{5}{3}0$, $E=60000$, $\frac{c}{l}=1\frac{1}{2}$, $P=30666$

we have,

$$M_n=[255550(12-n)+83333(10\tfrac{1}{2}-n]n$$

whence we find the strains in,

$$Aa=Bb=\frac{M_1}{98}=\frac{3602713}{9.8}=367620,$$

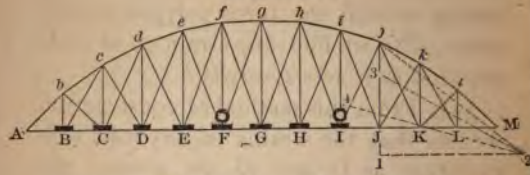
$$Ma=\frac{M_1}{8.43}=427000,$$

$$Cd=\frac{M_2}{17.3}=\frac{6527660}{17.3}=377320,$$

&c., &c., as entered in the table below.

140. *Maximum Web Strains.* The following method of ascertaining the max. web strains is due to Stoney ("Strains in Girders," etc., art. 211). Suppose the truss without weight. Let the live load, engines in front, extend to the foot of the tie whose max. strain is required. Then, in Fig. 16, if we suppose the panel IJ *ij* cut and forces applied at the cut pieces equal and opposite to the resistances of those pieces the right segment of the truss will be held in equilibrium by the reaction at the right

FIG. 16.




abutment, the horizontal tension in IJ, and the resultant of the strains in I*j* and *ij*. The two former met at M, hence the resultant at *j* must pass through M.

Therefore, if we draw, by scale, $\overline{ji}$ vertically, and equal to the reaction (which is also the shear over panel IJ), and draw 12 horizontally till it meets *j*M produced at 2, then $\overline{2j}$ is equal and opposed to the forces supposed *applied* in the directions, *ij* and *j*I at *j*; whence drawing 24 parallel to $\overline{ij}$, *j*4=strain on tie, and 24 on bow *ij* for this position of the load.

Similarly if we suppose the truss severed through I*jk*, 2*j* will be the resultant of the resistances of *j*J and *jk* at *j*; or, *j*2 acting in the direction from *j* to 2 *is in equilibrium* with those resistances.

Hence, drawing 23 parallel to *jk*, on following around the force polygon in the order, *j*23*j*, we find 3*j*=strain on *j*J *compression* as it acts *towards j*.

141. From this method of construction we see that the strains on any two web members as I*j* and *j*J are greater the greater the reaction, provided there is no load on the right segment; hence, omitting the case of the right segment being loaded for the present, the strains on a tie and the vertical connecting with its top, when the greater segment only is loaded are a maximum, when the live load—engines in front—extends from the *farthest abutment* to the foot of the tie. The counters J*i* and K*j* are supposed loose or out of action, hence were disregarded.

141. We proceed similarly for the other diagonals and verticals. The method is the same for the counters F$\bar{g}$, E$\bar{f}$, . . . . : the live load extending from their feet to the nearest abutment for their max. strains. Now add, with its proper sign (+ for compression, — for

tension), the effect on the *posts* of
dead load, as found from the const
tion Fig. 15, to the strains just fou
from Fig. 16, *when the live load exte*
*to the farthest abutment.*

| Post Fig. 16 | Dead Load. | Live Load. | Total Str |
|---|---|---|---|
| K*k* | − 11800 | + 13000 | + 12 |
| J*j* | − 11000 | + 22000 | + 110 |
| I*i* | − 11200 | + 27500 | + 163 |
| H*h* | − 11400 | + 31000 | + 191 |
| G*g* | − 11600 | + 32300 | + 207 |

Next, in Fig. 15 (1), conceive
diagonals only in the direction of
counters—they thus suffer compress
for a uniform load—and draw the
responding strain diagram due to d
load only. It is convenient to draw
directly over Fig. 15 (2), as the incli
tions of all the pieces (but the diagona
and also the loads, have been alre
laid off in proper position. An expla
tion of the construction for one pa
will suffice. Place *b* and *c*, Fig. 15
*on either* side of the diagonal from A

*ce*, so that the bow piece is M*b* and the chord piece B*c*. Then in Fig. 15 (2) extend *ab* to intersection with M*c*, and draw *cb* || *cb* (1), to intersection with B*b* prolonged; then *ab* = −18300 gives the strain in the first vertical due to dead load. Similarly we proceed for other panels. The strain diagram for Fig. 20, inverted, applies here exactly. The strains in the counters due to dead load are obtained from the same figure.

Since, when the live *load extends to the nearest abutment* the counter connecting with the post is alone in action, we must add the strains on the posts found from Fig. 16, for this case, to the strains just found on the diagonals in the direction of the counters, due to dead load. We thus find,

| Posts Fig. 16 | Dead Load. | Live Load. | Total Strain. |
|---|---|---|---|
| K*k* | − 17800 | + 16500 | − 1300 |
| J*j* | − 17000 | + 23000 | + 6000 |
| I*i* | − 16400 | + 27500 | + 11100 |
| H*h* | − 15300 | + 31000 | + 15700 |
| G*g* | − 14000 | + 32300 | + 18300 |

On comparing this table with the preceding, we see that, in this example, the posts are most strained when the live load extends from the farthest abutment to the foot of the tie connecting with them. The same is true for the main ties, since they can only take tension. The max. strains thus far found are entered in the following table :

142. We have previously found that for dead load the diagonal web members are only slightly strained. If the counters are tightened too much, it would tend to relieve the main ties of strain, but to increase the strains on the posts. Thus if counter *j*K is in action, say it is strained 10000 lbs., then *j*2 acting from *j* towards 2 is opposed to the the resultant of the resistances J*j*, *j*K and *jk*. Hence extend 23 so that 3 will have such a position that a line drawn from it parallel to counter *j*K to intersection, 5, with *j*J will measure 10000 lbs. The force polygon formed, *j*235*j*, gives as explained before, 23, compression on *jk* (*greater* than before), 35, tension on

counter (10000 lbs.) and 5*j* compression on post J*j*, considerably greater than before.

Hence an ignorant tightening of the counters may easily double or treble the strain on the posts for which they will be designed in what follows.

Another very objectionable feature in this truss, is the fact that for a uniform load there is tension only in the verticals, whereas, when the train is only partially on the bridge, they are each in turn subjected to compression; changing thus quickly from a possible 45000 lbs. tension to a max. compression of 20000 lbs. (about, for middle posts), or the reverse. The verticals were consequently designed, each to consist of two plates connected by the usual latticing and angle irons of sufficient section to resist both strains, and were of course "hinged at both ends."

143. We thus see that the great saving in the web in this form of truss is really the greatest objection to it, at least for a railway bridge. The bow form is best used in the plate girder.

144. The successive *reactions* at the right abutment in last Fig. are easily found from *eq.* (5), art. 19, by making the dead load, $p=o$. The reactions are the same then as the shears on the right segment. Expressing them in hundred weight, we have

| | | |
|---|---|---|
| $S_1=1391$, | $S_5=664$, | $S_9=158$ |
| $S_2=1188$, | $S_6=516$, | $S_{10}=92$ |
| $S_3=1000$, | $S_7=383$, | $S_{11}=39$ |
| $S_4=825$, | $S_8=264$, | $S_{12}=0$ |

Due regard was paid to the rear engine moving off the bridge by modifying the formula as suggested in art. 19.

145. Combining the max. strains found on the main ties due to live load with those previously given due to dead load (art. 137) the results are entered, together with the strains on the other members of the bridge in the adjoining table; from which we deduce, as before, the

BILL OF MATERIALS.

Bow String Through Bridge, 200′ span, 30′ rise.

| | lbs. |
|---|---|
| Bow and posts, with latticing &c.. | 83526 |
| 20 p. c..... | 16705 |
| Chord ties and counters..... | 80453 |
| 15 p. c..... | 12068 |
| Other items as in art. 47..... | 134100 |
| Total weight of bridge..... | 326852 |
| Assumed weight..... | 336000 |
| | 9148 |

146. We have assumed the same percentages and estimates of loops, transverse bracing &c., as previously given for the Triangular and Whipple bridges, which is only approximately correct.

The bow piece can be more conveniently constructed of some other form than the phoenix column, so that the above estimate is favorable to this form of truss. The principal objection urged to the Bow String Girder is, that the web members are so slightly strained from dead load, that the rolling load may find them out of action, thus giving rise to hurtful vibrations.

The difficulty in cross-bracing the bow is also a great objection to this truss as a through bridge.

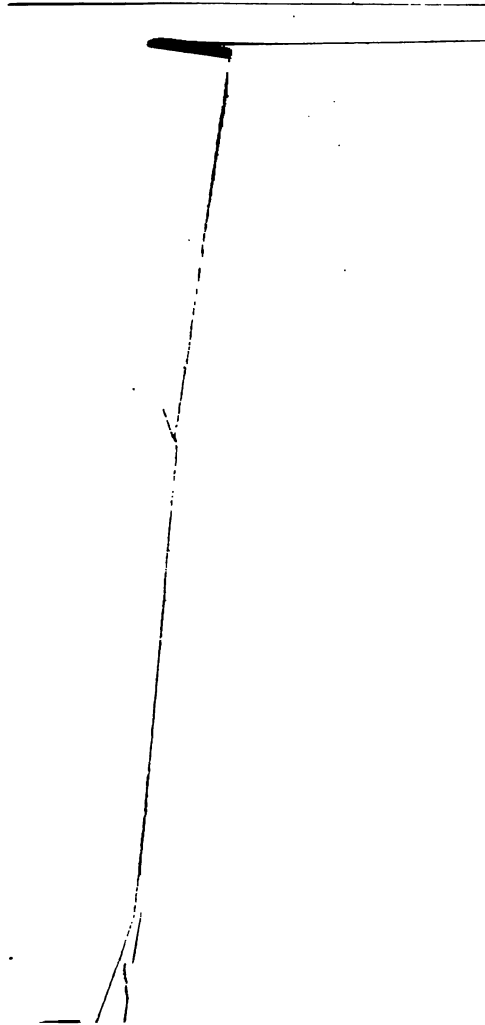
147. In the previous investigation of the Bow String Girder, only one system of triangulation was assumed and the most economical height was not found as for the other trusses examined.

As the web is comparatively light and some of the ties have about their most economical inclination, 45°, it is not probable that a double system of triangulation, for the span and panel length assumed, would involve as much saving as we found in the quadrangular truss.

The height assumed, 30 feet, is slightly greater than for the beam trusses, as it was thought that the light web would admit of a greater height. Trial only can determine the most economical height. It would seem that the change of a foot or so would effect a very slight saving however, if we may judge at all from the previous investigations concerning the beam trusses.

| Piece. | $d$ | $\frac{l}{d}$ | th. | Strain. | $\theta$ | $b$ |
|---|---|---|---|---|---|---|
| | ″ | | ″ | | | |
| A*b* | 13½ | 17.7 | 1 7/16 | 427000 | .39 | 800 |
| *bc* | " | 16. | | 413140 | " | 904 |
| *cd* | " | " | | 404370 | " | " |
| *de* | " | " | | 396330 | " | " |
| *ef* | " | " | | 388780 | " | " |
| *fg* | " | " | 1⅛ | 381660 | " | " |
| *PostK*k* | 5 | 33⅓ | ½ | 1200 | 0 | 300 |
| J*j* | 7 | | 9/16 | 11000 | 0 | " |
| I*i* | 8 | | 11/16 | 16300 | 0 | " |
| H*h* | 9 | | " | 19100 | 0 | " |
| G*g* | 9 | | " | 20700 | 0 | " |
| Latticing, angles, & | c. 27 lbs. pr. ft. | | | | | |
| Lower Chord | | | | 2263400 | .39 | 1042 |
| Tie K*l* | | | | 34200 | 0 | 750 |
| J*k* | | | | 41000 | 0 | " |
| I*j* | | | | 46000 | 0 | " |
| H*i* | | | | 49900 | 0 | " |
| G*h* | | | | 51300 | 0 | " |
| Counter F*g* | | | | 50200 | 0 | " |
| E*f* | | | | 48000 | 0 | " |
| D*e* | | | | 43200 | 0 | " |
| C*d* | | | | 43100 | 0 | " |
| B*c* | | | | 44200 | 0 | " |
| Vertical Ties | | | | 45000 | .13 | 847 |

* The thickness of metal "th" is made up of that due to acting as a strut.



148. *Analytical Formulæ.*

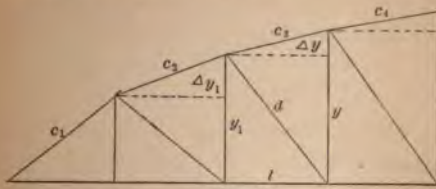



Fig. 17

Let us call the successive lengths of the bow, $c_1, c_2, \ldots$ ; the length of a post $y$, the length of the post to left of it $y_1$ ; $d$ the length of a diagonal connecting $y$ and $y_1$; $\Delta y_1$ = difference between $y_1$ and the length of the post to the left of it ; $\Delta y = y - y_1$. Also the pieces can be designated by the letters next them on the figure above, prefixing the words *post*, *tie* or *piece* to avoid any confusion.

Call the *moment* of the external forces (reactions and loads) on one side of *post* $y$, about its foot as a center of moments, $M_y$ ; similarly for $M_{y_1}$; also call the *shear* in the panel included by $y$ and $y_1$, *for the same distribution of the load* $S_y$.

Designate a panel length by $l$ ; th

inclinations of pieces $c_1$, $c_2$, . . . by $\alpha_1$, $\alpha_2$, . . . and of the diagonal $d$ by $\beta$.

Also denote the strains in $c_3$, $y$, $d$, etc., by the corresponding capital letters, $C_3$, Y and D.

149. Suppose a vertical section cutting pieces $c_3$ $d$ and $l$, and take the intersection of $d$ and $l$ as a center of moments to find the strain $C_3$, whose lever arm is

thus, $$y \cos \alpha_3 = y\frac{l}{c_3}$$

$$\therefore M_y = C_3 y \frac{l}{c_3} \quad \therefore C_3 = \frac{M_y}{y}\frac{c_3}{l} \quad . . . \quad (1)$$

If we supply forces (as in art. 7, Fig. 2) at the supposed vertical section (cutting $c_3$ $d$ and $l$) *equal and directly opposed* to the resistances of the cut pieces, we must have the algebraic sum of their vertical components equal to $S_y$. Compare art. 8. Now since the bow is always in compression—since every weight on the truss causes an upward moment—the external force opposed to the resistance in piece $C_3$ acts downwards.

$$\therefore S_y = D \sin. \beta + C_2 \sin. \alpha_2 = D\frac{y_1}{d} + \frac{M_y}{y}\frac{c_2}{l}\frac{\Delta y}{c_2}$$

$$\therefore D = \frac{d}{y_1}\left(S_y - \frac{M_y}{y}\frac{\Delta y}{l}\right) \quad . \quad . \quad . \quad (2)$$

On the right half of the truss the force opposed to C acts upwards. Since $\Delta y$ is then minus eq. (2) applies on changing the sign of $\Delta y$. Again, note that when $S_y$ is minus it must be so substituted in eq. (2). When D is plus, the diagonal is in tension, otherwise in compression.

The reader will do well to sketch the truss up to the section and the forces acting on it, as in Fig. 2, in this and subsequent articles.

150. Next conceive the section parallel to diagonal $d$, cutting pieces $c_2$ $y_1$ and $l$; and balance the vertical components of the "acting forces," which include the reaction and loads left of the section, and the supposed forces equal and opposed to the resistances of the cut pieces

$$\therefore S_y = Y_1 + C_2 \sin. a_2$$

From eq. (1) we can derive,

$$C_2 \sin. a_2 = \frac{M_{y_1}}{y_1} \frac{c_2}{l} \frac{\triangle y_1}{c_2} = \frac{M_{y_1}}{y_1} \frac{\triangle y_1}{l}$$

Now $M_{y_1} = M_y - S_y\, l$, as may be shown as follows:

Suppose a section cutting $y$, $c_3$ and the chord piece to right of piece $l$; and supply forces equal and opposed to the resistances of the cut pieces. Decompose the applied force $= C_3$, at top of post $y$, into vertical and horizontal components. The latter component is equal to the force applied at the cut chord piece, and forms with it a left-handed couple equal to $M_y$, since the moment of the vertical component $+Y = S_y$ is zero; these forces acting through the center of moments—the foot of post $y$.

Hence, regarding the left-handed couple $(= M_y)$ and the sum of the vertical components $(= S_y)$, acting downwards if $S_y$ is $+$, as the external forces at the section, we have the *moment* about the foot of post $y_1$, $M_{y_1} = M_y - S_y l$ as was to be proved.

It is useful to note that, since $M_y = M_{y_1} + S_y\, l$, the moment increases in going from $y_1$ to $y$, so long as $S_y$ is positive. Therefore at the point where $S_y = 0$ the moment is a maximum (compare art. 90).

Substituting the above value for $M_{y_1}$, we have for the strain in the post $y_1$

$$Y_1 = S_y - \frac{M_y - S_y\, l}{y_1} \frac{\Delta y_1}{l} \quad . \quad . \quad . \quad (3)$$

As before, when $S_y$ or $\Delta y_1$ are negative, they must be so substituted in this formula.

151. *The Maximum Strains* on a tie we have previously found to be when the load extends from the foot of the tie to the farthest abutment; for a counter tie, the load extends from its foot to the nearest abutment. Therefore, for maximum strains, the shears $S_y$ are easily obtained from eq. (5), art. 19.

The corresponding moments $M_y$ are found thus : to $S_y$ add the dead load between the post marked $y$ and the left abutment to find the *reaction*, whose moment *about* $y$, minus the moment of

the downward loads from the abutment to $y$ gives $M_y$.

152. *Example.* Required the max. strain, D the counter $\overline{iJ}$ ever sustains, for the bow string girder, Fig. 16 previously examined. The live load extends from M to J. By table, art. 21, $S=-19172$. Reaction at $A=-19172+8\times14000=92828$.

$$\therefore M_J=92828+150-112000\times75=5,524,200,$$

whence by eq. (2)

$$D=\frac{31.8}{26.9}\left(-19172+\frac{5524200}{23}\frac{3.9\times6}{100}\right)=43770.$$

The graphical analysis gave 43200.

153. For a post as $\overline{Ee}$, the live load must either extend from M to F or from A to D to cause max. strains. In the first case the strain on $\overline{Ee}$ is by eq. 3.

$$Y_1=87364-\frac{9613667-87364\frac{19}{6}}{26.9}\cdot\frac{3.9\times6}{100}=16402$$

Secondly, since post $\overline{Ii}$ for live load from M to J sustains the same strain as $\overline{Ee}$, when the load extends from A to D, we have for the strain in this case,

$$Y_1 = -19172 + \frac{5524200 + 19172\frac{100}{6}}{26.9}.$$

$$\frac{2.3 \times 6}{100} = 10808$$

The graphical analysis gave 16,300 and 11,100 respectively. In this way we can form the following table; where column 1 gives the post, columns 2 and 3 the strains sustained by it, when the live load extends from the foot of the tie or counter connecting with it to the farthest or nearest abutments respectively:

| | | |
|---|---|---|
| *Cc* | 1600 comp'n | 182 tension |
| *Dd* | 9318 " | 6200 comp'n |
| *Ee* | 16402 " | 10808 " |
| *Ff* | 19008 " | 17150 " |
| *Gg* | 18470 " | 18470 " |

In this case, we see that the posts are most strained when the live load extends to the farthest abutment. The strains

found by the preceding formulae were found to differ from those found by construction less than 1000 lbs. as a mean, the extreme difference being 3000 lbs. This is due partly to the short lengths of the bridge members from which their inclinations were derived, and partly from the unavoidable errors of construction, as well as from the dimensions being taken to only tenths of a foot in the formulae. Other formulae could be given but we have preferred those of Schwedler on account of their compactness and as introductory to his bridge.

154. *The Schwedler Bridge.*—In this bridge, the upper chord is parallel to the lower chord in those panels where counters would be required in a Pratt or a Howe truss; *e.g.*, for the four middle panels, for the span, panel length and loads previously considered. In the other panels, the height of post is so regulated that no counters are required; *i.e.*, the diagonals act as ties only or as struts only.

Now the max. reverse strain in a

Fig. 18

diagonal, as $\overline{Dc}$ (Fig. 18), is when the front engine is at C and the load extends to A. The condition for this truss is, that this strain must be zero for those panels where no counters are to be used. Placing *eq.* (2) equal to zero, we deduce, for the left half of the truss,

$$\triangle y = y\frac{S_y\, l}{M_y} \quad \ldots\ldots\ldots (4)$$

In this formula, it is understood that the front engine is at the post marked $y_1$, and the live load extends to the left abutment, the moment $M_y$ being taken about the post $y$.

155. *Example.*—Take the span=200′, $l=\frac{100}{6}$, loads, &c., as before. Assume $Ee=30'=y$.

With engine at D and load extending to A, the moment about E is

$$M_y=5{,}844{,}000 \qquad S_y=-19172$$

160. The results for the Schwedler truss of 200′ span 30′ center height, &c., are entered in the following table:

BILL OF MATERIALS.

Schwedler Bridge, (through) 200′ span 30′ high.

| | lbs. |
|---|---|
| U. chord and posts | 78,574 |
| 20 p. c | 15,714 |
| Ties and lower chord | 65,801 |
| 15 p. c | 9,870 |
| Other items as in art. 47 | 134,100 |
| Total weight of bridge | 304,059 |
| Assumed weight | 336,000 |
| | 31,941 |

161. Collecting together the estimates of weights of the *through* bridges examined, arts., 108, 128, 145 and 160, we have, in round numbers,

| | | | | |
|---|---|---|---|---|
| Weight of | Triangular | Truss, | 325000 | lbs. |
| " " | Whipple | " | 325000 | " |
| " " | Pratt | " | 333000 | " |
| " " | Fink | " | 356000 | " |
| " " | Bow String | " | 327000 | " |
| " " | Schwedler | " | 304000 | " |

*The pier* towers of the Fink were put at

| | b. | A |
|---|---|---|
| | 7650 | 4 |
| | 9040 | 3 |
| | " | 3 |
| | " | 3 |
| | " | 4 |
| | " | 4 |
| | 3930 | |
| | " | 1 |
| | " | 1 |
| | " | 1 |
| | 2810 | 1 |
| | 8470 | |
| | 7500 | 1 |
| | " | 1 |
| | " | 1 |
| | " | 1 |
| | " | 8 |
| | " | 4 |
| | " | 2 |
| | 0420 | 21 |
| | " | 25 |
| | " | 29 |
| | " | 33 |
| | " | 35 |

Phœnix c
s taken as
h ends "

27000 lbs. These weights will differ still more on a re-estimate, on assuming dead loads more in accordance with the truth.

The superiority of the Schwedler Truss in point of weight is marked, and should receive careful attention from constructors.

162. Several other through trusses were examined, as the Lenticular or Fish-bellied Girder, and the Triangular, with two suspenders to a panel, instead of one; but little or no economy was found over the Whipple Truss above. The truss figured in VAN NOSTRAND'S MAGAZINE, for November, 1877, p. 461 (Fig. 8), on data above, weighs 320,000 lbs.

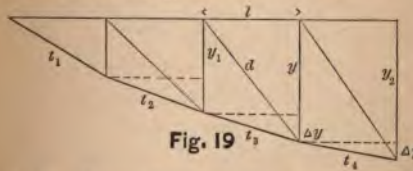



Fig. 19

163. BOW STRING DECK BRIDGE.—*Formulae.*—Call the bow pieces $t_1, t_2 \ldots$; the length of the post to the right of $y_1$

$y_2$; put $y_2 - y = \triangle y_2$; the other notation as before.

The strain in piece $t_4$ is, $T_4 = \frac{M_y}{y} \frac{t_4}{l}$ ..... (5), since the lever arm of piece $t_4$ about the top of post $y = y \frac{l}{t_4}$.

164. Now conceive a vertical section cutting pieces $l$, $d$ and $t_3$, and balance the vertical components.

$$\therefore S_y = D\frac{y}{d} + T_3 \frac{\triangle y}{t_3} = D\frac{y}{d} + \frac{M_y - S_y l.}{y_1} \frac{t_3}{l} \cdot \frac{\triangle y}{t_3}$$

$$\therefore D = S_y \frac{d}{y} + S_y \frac{d}{y} \frac{\triangle y}{y_1} - \frac{d M_y}{y_1 y} \cdot \frac{\triangle y}{l}$$

$$\therefore D = \frac{d}{y}\left( S_y - \frac{M_y}{y} \frac{\triangle y}{l} \right) ..... (6)$$

To the right of the center, $\triangle y$ is minus, whence the $-$ sign in the ( ) is changed to $+$ as is evidently correct.

165. Next conceive a section parallel to piece $d$, cutting pieces $l$, $y$ and $t_4$, and *balance* the vertical components of the

forces opposed to the resistances of the cut pieces with $S_y$ we find,

$$Y = S_y - T_4 \frac{\Delta y_2}{t_4} = S_y - \frac{M_y}{y} \frac{\Delta y_2}{b} \ldots\ldots (7)$$

The same formula applies to the right of the center, since $\Delta y_2$ is then minus and the eq. becomes

$$Y = S_y + \frac{M_y}{y} \frac{\Delta y_2}{l}, \text{ where } \Delta y_2 = y - y_2$$

When $S_y$ is minus it must be so regarded in the previous equations.

As the application of the formulae is essentially as just explained for the through bridge it is needless to give it.

166. As it was of special importance to ascertain whether the posts were most strained by the live load extending to the farthest or nearest abutment from the posts considered, for the loads, &c., as previously given, the following strains were tabulated referring to the next figure:

| Post. | Live Load from Post to farthest abt. | Live Load from Post to nearest abt. |
|---|---|---|
| *ab* | 50718 | 60655 |
| *cd* | 57400 | 60584 |
| *ef* | 66060 | 62970 |
| *gh* | 70108 | 68450 |
| *ij* | 71564 | 71750 |
| *kl* | 73632 | 73632 |

from which we ascertain that *in this case* $\overline{ab}$ and $\overline{cd}$ are most strained when the live load extends to the nearest abutment; for the other posts it extends to the farthest abutment.

167. In this deck truss the constructive difficulty of the joints of the bow is not experienced as the bow is not in tension. The web is heavier and the posts may be made of phoenix columns, with *square* joints at their connections with the chord. These vertical members now bear but one kind of strain; altogether the truss is a good one, and is worthy of more consideration than it has received, as it will be found to be the *most* economical in weight of any of the *deck* bridges examined.

168. *Graphical Analysis*—The next figure gives the form of the truss, omitting the counter ties, and the strain diagram for the uniform dead load of 14000 lbs. per panel; which, using Bow's admirable notation, needs no further explanation. From the strain diagram we find the dead load strains on ties, *bc*, *de*, *fg*, *hi*, *jk*, respectively, to be, in round numbers, 6000, 5000, 4000, 3000 and 2000 lbs; the strains on the posts vary from 18000 on *ab* to 14000 lbs. on *kl*.

169. If the members of the *bow* in any bow string truss are given such inclinations that the strains on the web ties are zero, *i. e.* that in the strain diagram *b* and *c* coincide, as well as *d* and *e*, *f* and *g* &c., then the apices of the bow are points in a parabola. For in the *strain diagram* the points *a*, *b*, *c*, *d*, *e*, . . . . will all lie in the vertical through *a*, and then since $ab = bd = dg \ldots$, the *difference* between the *tangents* of the inclinations to the horizontal of any two consecutive bow pieces is the same. For regarding momentarily A*a* as unity, $ab = bd = dg = \ldots$ is the tangent difference in question. This is a property of the parabola. Thus assume its equation $y^2 = mx$; give $x$ the increment $h$ and call the corresponding increment of

$y$, $k$ . $\therefore (y \times k)^2 = m\,(x \times h)$. Whence, expanding and taking the difference between the two equations, $2\,yk + k^2 = mh$

$$\therefore \frac{h}{k} = \frac{1}{m}(2y + k)$$

Now $\frac{h}{k}$ = tangent of the angle made by a chord of the parabola with the axis of $y$. (In the figure of the truss a horizontal drawn through the point $Mjm$ (the origin) may be taken as the axis of Y, the line $kl$ as the axis of X).

Now giving to $y$ the successive values $o$, $k$, $2k$, $3k$ . . . . we find for $\frac{h}{k}$ the successive values,

$$\frac{1}{m}k, \frac{1}{m}3k, \frac{1}{m}\ 5k, \frac{1}{m}7k, \ldots .$$

whose difference is $\frac{1}{m}2k$, *a constant*, which was to be demonstrated.

In the figure of the truss $k$ may be regarded as equal to a panel length $16\frac{2}{3}$ feet, whence $h$ will be the vertical distance between the extremities of the bow piece, the tangent of whose inclination to the horizontal is given by the value of $\frac{h}{k}$, corresponding to the value of $y$ for the lower end of the bow member.

When a parabolic bow string is loaded uniformly the strain throughout the string is

uniform, since, in the strain diagram $a$, $b$, $c$, $d$ . . . are in the same vertical.

170. As before, to find the max. strains due to live load only on tie 14, we suppose the front engine at IJ and the train extending to the farthest abutment. Lay off the reaction at LM from 1 to 2, and draw 23 parallel to chord to intersection with 13 passing through LM. Then 13 =resultant of reaction and strain in

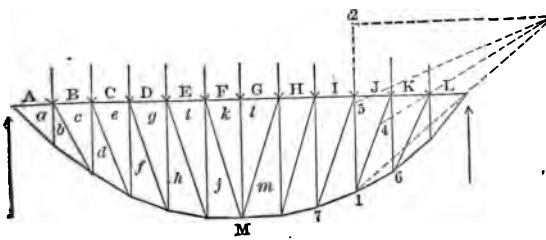
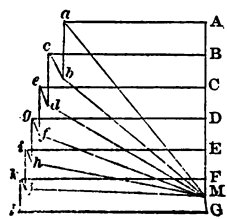



FIG. 20.



chord panel J, must be in equilibrium with strains in tie 14 and 16; hence drawing 34 parallel to 16, 14=strain on tie due to live load.

Similarly, drawing 35 parallel to 17, 15=strain on post 15 *due to this position of the load;* but we shall find that the posts are most strained *from the live load alone* when it extends from the *nearest* abutment to the post considered; which strains as well as the strains on the counters are determined in a similar manner to the above. Now add the post strains due to live load—extending to farthest abutment—to dead load strains obtained from diagram above, and compare these totals with those found, by adding the post strains due to live load —extending from post to nearest abutment to those found (from a diagram similar to Fig. 15 (1) and (2) inverted), due to dead load, from a figure where diagonals in the direction of counters are alone represented.

We reach the conclusion of art. 166.

From the last mentioned diagram, the dead load strains on counters are found.

171. The total strain on any post cannot be less than 45000 lbs., (the panel reaction), when the engine is directly over it; and proceeding as above we find that the strain on the posts due to live and dead load is not in any case less than 45000 lbs.

Suppose that the loads are so placed that AB has, say, double its value given in the figure, the reaction MA remaining the same; then in the strain diagram for $a$B, $ba$ AC$c'b$ ($c'$ being in the prolongation of $bc$ at its intersection with C$e$) we see that $bc$ must now act as a strut; *otherwise* $ab$ takes the whole load (2 AB) and a counter must be introduced from its foot to $c$C.

It follows that, for the assumed truss, the counters next an engine *may* be in action when the whole bridge is loaded.

172. *The maximum chord strains* are, for panels A$a$, B$c$, C$e$, . . . . respectively,

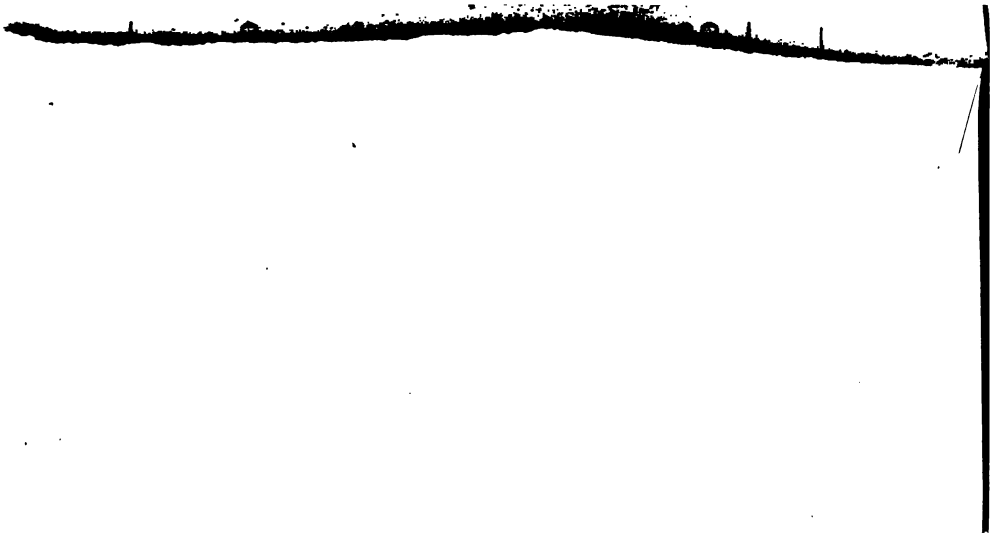
$\frac{M_1}{9.8}, \frac{M_2}{17.3}, \frac{M_3}{23}$ ....; the lever arms
ing respectively $\overline{ab, cd, ef}$, ....

For the Bow we ascertain the le
arms as follows: conceive the panel
M*dec* cut, take the intersection of
tie and upper chord panel *c*C as a cen
of moments, and from it draw a p
pendicular to M*d* produced, which
thus the lever arm of the strain in
In this case $M_2$ divided by this lever a
is the strain in M*d*, $M_2$ being the ma
mum moment when *c*C is taken as
center of moments.

The max. moment for both M*a*
M*b* is $M_1$; hence $M_1$ divided by
length of perpendiculars from *a*B to
and M*b* respectively give the strains
M*a* and M*b* respectively.

Similarly for other divisions of
bow.

173. The strains are entered in
following table. For the posts, $\theta$
taken at .25 as an average, from wh
none of them differ much. For the
$\theta$ is put at 0. The maximum ch



strains are $Aa$=367620, $Bc$=377320, $Ce$=381510, $Dg$=384540, $Ei$=384790, $Fk$=381660; the sum of the last five being 1,909,820, as entered in the table. The total weight found is the same whether these chord panels are considered separately or collectively, the length being the same for each piece.

BILL OF MATERIALS.

Bow String Deck Truss, 200′ span, 30′ center depth.

| | lbs. |
|---|---|
| U. chord | 54800 |
| Posts | 19853 |
| 20 p. c. on two last | 14930 |
| Bow | 54494 |
| Ties | 24235 |
| 15 p. c. on two last | 11809 |
| Other items (art. 91) | 127230 |
| Total weight | 307851 |
| Assumed weight | 336000 |
| | 28,649 |

174. The *Triangular*, Fig. 7 as a Deck Truss, can be best compared with the Whipple, etc., with vertical end posts. However, let us assume the abutments

built up to grade to compare with the deck trusses examined. The max. strains are identical with those for the through bridge, except that the vertical posts must now sustain a max. load of about 45000 lbs. The suspenders may be made of one square inch cross section. Estimating as usual we find its weight as given below:

175. *Comparison of Deck Bridges of 200′ span*, loads, etc., as previously given.

| | | | |
|---|---|---|---|
| *Triangular*, | 28′ high, | 321,000 | lbs. |
| Whipple, | " " | 326,000 | " |
| Fink, | 33⅓ " | 317,000 | " |
| Bow String, | 30 " | 307,000 | " |

The Schwedler as a Deck Truss would doubtless prove lighter than any of the previous trusses.

176. With other details than those assumed—and our best bridge companies have devised some excellent ones—the results found may be slightly varied; but it is believed that the general comparisons are correct for any given details.

At any rate the data is all given so that errors can be detected or modifications of design readily made.

177. Although the loads previously assumed are for railroad bridges, yet the formulae, or methods given, can be easily adapted to highway bridges, where the live load is usually taken as so much per square foot of roadway, varying from 35 to 100 lbs. per square foot.

178. A matter of great interest to engineers is the determination of good formulae for compression members. Government aid is anxiously looked for in this direction to institute the proper experiments. Various formulae for different cross-sections are being introduced in some specifications, though they are founded on comparatively few experiments, and thus are only provisional, as indeed are the formulae previously used in this paper for unit strains; but the deduction of Wöhler that a piece will bear a smaller maximum strain the greater the extremes of strain to which it is subjected is not provisional, but a

fixed fact, which must be regarded if a bridge is to be designed scientifically; and it is to be hoped that the above will show that there is no difficulty in the application of Launhardt's formula founded on this law.

179. It is believed that there will be no difficulty in applying the preceding principles to any form of truss in ascertaining the greatest or least strain that any member is ever called on to bear. If the truss is composed of two or more web systems, not connected at their intersections, estimate the influence of each separately and combine the effects for a piece that is common to the two or more systems. All the principles relating to the method of ascertaining max. and min. strains, etc., that pertain to a simple system apply to each web system in turn.

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